Dynamic Contracting for Development Aid Projects: 
Mechanism Design and High Performance Computation

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Abstract

Developing economies share both microeconomic and macroeconomic characteristics which are often unique relative to their more developed counterparts. Indeed, many authors (e.g. Parente and Prescott 2000) have emphasized the role of institutional frictions within developing nations as a major determinant of economic growth (or the lack thereof). We examine one type of institutional friction, concerning the observation and reporting of information, and construct a straightforward dynamic contracting model of foreign donor investment in an aid project. We show that even within a simple class of such models, the dynamic contracting problem rapidly becomes computationally intensive. We argue that the natural modeling, simulation and testing environment to both analyze development aid issues and help generate effective aid policy should involve—indeed, rely upon—high performance computational resources to solve such dynamic contracting problems.

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1 Introduction

In development economics it is often the case that a foreign aid entity—a development bank, private investor, or other governmental/non-governmental institution—wishes to engage in direct investment in a host nation—usually represented by a government—for e.g. infrastructure improvements, public safety and health investment, education, etc. Unfortunately, in the real world both the foreign aid donor and the aid recipient suffer from two-sided asymmetric information, in which the motivations and even the actions of the two participants are unclear. In addition, the outcome of investment may be obscured by factors external to participant behavior (such as third party activities, informal sector frictions, or local security issues), making it difficult to attribute specific investment outcomes to specific behavior.

In such environments it is difficult, if not impossible, to arrive at an adequate ad hoc solution to this problem—but as development aid projects are certainly created and implemented all the time, there must exist some mechanism (or class of mechanisms) for the two participants to trust each other enough to invest sometimes sizable funds into development projects. We take as our point of departure the assumption that such a mechanism is a contract between a foreign development aid agency and a recipient who represents the locality where investment takes place. It stands to reason that examining this relationship using the lens of contract theory may shed light some light upon the subject, and this paper is an attempt to examine the benefits (and limits) of contract theory as applied to economic development investment problems.

Kilby (2001) identifies the principal-agent contracting relationship as a possible description of the connection between a donor (here, the World Bank) and an aid recipient, but does not provide a formal analysis. A formal treatment of the theory of contracts applied to development aid, in a static framework, is given in Azam and Laffont (2003). By tying the contract’s payoff structure to the consumption level of the local population, they
claim that such “conditional aid” is more effective than unconditional aid which is not tied in this fashion. By contrast, Svensson (2003) replies that in a similar class of static contract models, conditional aid may not be an effective aid deployment mechanism.

We propose a contracting model which is dynamic rather than static, in order to capture the realistic situation that when donor aid is used for investment, that investment takes time, and time introduces problems of commitment which cannot be captured in a static framework. It is this dynamic feature of investment, moreover, that allows for multiple opportunities for either donor or recipient to act in a self-interested manner. Such actions may, in the end, turn out to lessen the value of the final investment project. We interpret this outcome as exhibiting one facet of “aid ineffectiveness”.

The model is a dynamic Costly State Verification framework (see e.g. Townsend 1979) with stochastic monitoring, introduced by Monnet and Quintin (2005), and extended here to include not only unobserved output (which the agent can misreport), but also unobserved investment, so that the model also contains a hidden action component. Having investment hidden and imperfectly correlated with output, allows us to model institutional frictions (such as bureaucratic inefficiency, informal sector frictions, poor accounting standards, etc.) as informational asymmetries. Moreover, such informational asymmetries could be bilateral where development aid is concerned—while most of the literature has concentrated on models where the donor is (explicitly or implicitly) the principal and the aid recipient the agent, it is certainly possible that the aid recipient is unsure of the mechanism by which aid propagates from the donor to the final investment project, and hence has features more in common with principal than agent in that case (Buse 1999), (World Bank 2005).

In this paper we treat a simpler case of single-sided uncertainty, as we would like initially to present and analyze the dynamic contracting problem in a step-by-step manner before adding more realistic extensions with future work. In order to highlight the possi-
bility that it is the donor aid transmission network which might be hidden from view, we present the model with the aid recipient as principal and the donor as agent. Although this may seem provocative, in reality the fight to eliminate fraud and corruption from the funding supply side is one of the main efforts of multilateral development agencies in combating corruption in general. For example, consider the President’s Forward to the World Bank’s 2005 *Annual report on investigations and sanctions of staff misconduct and fraud and corruption in Bank-financed projects* (World Bank 2005):

While much has been accomplished, much more remains to be done. On the internal side, continued vigilance is needed to ensure that the Bank’s own house is in order. Bank staff must be beyond reproach in their personal and professional conduct. In terms of the Bank’s lending activities, the diversion of funds from development projects through fraud and corruption, when it occurs, directly injures the ability of the Bank, its partners and its borrowers to achieve the goals that have been set for poverty reduction. Resources lost to fraud and corruption are an unacceptable drain on development effectiveness, not to mention the damage to the credibility of lending institutions such as the World Bank.

But it is not just the financial damage from fraud and corruption that should be of concern to us. It is the fact that corruption sets in motion a chain of events that can wreak havoc on a development project. The money to pay a bribe must come from some part of the project; as a result, prices may be raised, and/or quality and performance lowered. Less qualified bidders win by bid rigging while qualified bidders become discouraged and stop bidding. In addition, citizen awareness of unchallenged corruption undermines trust in government and public institutions, which leads to acquiescence to poor quality and performance in public services and infrastructure - and to an unwillingness to report fraud and corruption. All of these effects must be considered when we assess the true impact of corruption on publicly financed projects. (p. 4)

At the end of the day, both donor and recipient wish fraud and corruption to be eliminated—the difficulty is in observing the trail of funding from source to destination, which implies that both supply side and demand side suffer from incomplete information. As a first stroke toward analyzing this double-sided uncertainty, we limit our analysis to
the supply side of the problem. Eventually, of course, future research must concentrate on the double principal-agent problem the realistic situation demands.¹

We assume, then, that the donor invests an amount each period into a project that is unobservable by the recipient, and that the donor observes the realization of investment (or ‘output’ of the project), while the recipient does not. The recipient’s goal, then, is to design a contract whereby the donor chooses to fully invest in the project, and report output truthfully. The tool which the recipient uses to enforce incentives is stochastic monitoring or auditing (see Border and Sobel 1987), coupled with a menu of contingent transfers and penalties. Conditional upon reported output, the recipient can pay a fixed audit cost and observe realized output and penalize the donor in the event that the donor has lied. This reproduces the real-world condition that it is possible in principle for an expert to observe the outcome of investment into a development project. In most cases of infrastructure, hospital and school investment this is plausible, although for more specialized projects (e.g. internet backbone or broadband investment) the output of investment may be more difficult to measure.

The possible underinvestment by the donor is, however, never directly observable. Rather, we suppose that the recipient must form expectations over whether or not the donor is underinvesting, and that the recipient updates these expectations in Bayesian fashion whenever the donor is audited and output is observed. Thus, the transfers and penalties which comprise the contract are dependent upon the expectations of the recipient, and for some expectation values an optimal contract will not exist.

We follow Monnet and Quintin (2005) and restate the contract as a dynamic programming problem, which is based upon the ‘continuation value’ state variable technique of Spear and Srivastava (1987). Due to unobserved investment and the Bayesian prior specification, a full analytical treatment of the model is not possible. But this does not

¹We are grateful to Jim Cassing for his insights and comments regarding the role of intermediaries and contractors on the supply side of donor aid.
obviate its use, as we turn to computational analysis to provide solutions to the dynamic program. Solving the system numerically is a non-trivial task—there are many incentive compatibility conditions which must be checked, and the resulting optimization takes place in a high number of dimensions. Fortunately, parallel and distributed computing resources exist which can, in principle, divide the overall problem into a collection of smaller problems which may be solved individually, and then recombined. In order to take advantage of such high-performance computing resources, we solve both the standard model of Monnet and Quintin (2005) and our extended “underinvestment model” to provide benchmarks for scalability. We utilized standard ‘brute force’ grid search on a cluster at the National Center for Supercomputing Applications (NCSA) at the University of Illinois at Urbana-Champaign to obtain these benchmarking results, which are presented in detail in Section 3.

Finally, it is worth noting that solving dynamic contracts numerically has implications for contract design and development policy, as well as demonstrating the application of contract theory specifically to development economics. Since the optimal contract can be found given an initial set of data, it should be possible to evaluate both existing development investment projects ex post and also new and planned development projects. Thus, this procedure becomes a valuable tool to both understand the contracting problem which exists between donor aid agencies and host nations, and also to propose an optimal contract (or set of contracts?) for a given environment.

One might balk at the suggestion that policymakers are capable of using such complex computational modeling to understand and implement policy. But this would be unfair. Modeling in general involves decisions about 1) the variables to be included, 2) their hypothesized effects, 3) the choice of general functional forms to represent the variables and how they interact, 4) the mathematics of equilibria and comparative statics and finally 5) the numerical solutions of such models. Policymakers are typically just as aware
as research economists of 1) and 2), while they can be quickly introduced to 3). We move closer to achieving the goal of a ‘democratic methodology’, which bypasses 4) and 5) as much as possible, by making the mathematical analysis and numerical simulation as painless as possible—this is accomplished by standardizing the contracting selection problem.

We argue that one way to get policymakers to appreciate the benefits of computational economics is to allow them to model their actual heterogenous constituents—the ultimate recipients of aid—and help to visualize the effects of various policies both for overall welfare and for the possibility of continuing as a policy-making authority (e.g. by analyzing the effect of policy on voting). The key challenge is, of course, heterogeneity, whether of agents or of technologies, and this continues to be viewed as an almost impassable stumbling block. The fact that people are different is the most banal of everyday observations, and if we have the goal of developing a methodology that will allow ordinary discourse a role in the policy process, the ‘democratization of technique’ becomes a necessity.

We thus wish to begin with a framework that will allow specific parameterizations to suit a particular development aid problem. The advantage of a computer based methodology is that it can bypass many questions of tractability and allow an economist or policymaker to directly impose restrictions on a model and check out the results. Policymakers, and their constituents, can be readily taught to visualize the restrictions that can be put into an economic plan.

The paper is organized as follows. Section 2 introduces the costly state verification dynamic contracting model with unobserved output and investment. Section 3 discusses the computation of the problem for two benchmarking cases, while Section 4 discusses extensions to the basic model which, we claim, are both realistic and feasible given our methodology. Finally, section 5 concludes, and provides avenues for future research.
2 The Model

The model is based upon the costly state verification framework of Monnet and Quintin (2005), which is based in turn upon a dynamic version of Border and Sobel (1987) and Townsend (1979).

There are two agents, a donor and a recipient. The donor embarks upon a development project in the recipient’s country, and the recipient receives the final benefit of the project when it is completed. For example, the project may be the construction of a road, an electricity transmission grid, a water distribution system, a school, a hospital, etc. The only restrictions on the project’s particulars are that it have 1) a well-specified construction time, and 2) a well-specified benefit to the recipient.

Time is discrete, and it is assumed that both the donor and the recipient take actions which are indexed by time. Because the project has a certain construction time $T < \infty$, the donor must invest in the project each period until it is completed. The investment level $I_t$ is assumed for simplicity to take two values, i.e. $I_t \in \{1, 1 - \gamma\}$. We interpret $I_t = 1$ as corresponding to the case where the donor fully invests in the current period. By contrast, $I_t = 1 - \gamma$ corresponds to underinvestment by the donor—the donor expropriates a fraction $\gamma \in [0, 1)$ for its own benefit, which may be thought of as e.g. a maintenance cost or ‘overhead’ cost of doing business in the recipient nation, or as the logistical cost of bringing a recipient’s aid proposal to the level at which it can be implemented (see e.g. Kilby (2001)). We assume that $\gamma$ cannot equal the total amount of possible investment in order to capture the notion that no investment at all would be immediately observable by both parties.

Investment $I_t$ is assumed to be translated into a realized investment, or ‘output’, level $i_t$ for the current period. One may consider this realized investment as that result of the investment transformation process which produces effects ‘on the ground’, i.e. that which can be considered as a measurable outcome of investment. While the investment
that the donor makes may be partially internal (and is, in any event, assumed to be unobservable by the recipient), the realized investment level \( i_t \) can in principle be observed by both parties. We assume for simplicity that there are only two possible output realizations, so that \( i_t \in \{i_L, i_H\} \forall t \), with \( i_L < i_H \).

In the real world it is generally not true that the donor’s investment outlay is immediately translated into a predetermined output level. Rather, there is some unavoidable randomness in the construction process which limits the actual impact of investment applied to the project—indeed, in a more well-developed version of the model, we would assume that the outcome of the project is jointly dependent upon actions of both the donor and the recipient. Here, we model this imperfect correlation between investment and output by assuming that the output \( i_t \) of investment \( I_t \) in period \( t \) is determined by an exogenously specified distribution of investment outcomes, \( f(i_t | I_t) \). To capture the notion that lower investment leads to lower realizations we suppose that:

**Assumption 1** *The probability \( f(i_t | 1) \) first-order stochastically dominates \( f(i_t | 1 - \gamma) \).*

Recall that first-order stochastic dominance implies that the probability distribution \( f(i_t | 1) \) lies everywhere at or below the probability distribution \( f(i_t | 1 - \gamma) \), which implies that the mean of \( f(i_t | 1) \) is higher than the mean of \( f(i_t | 1 - \gamma) \). Full investment by the donor will, on average and after many draws, lead to higher investment realizations. For the case treated here, where output \( i_t \) may take only two values \( i_H \) or \( i_L \), first-order stochastic dominance reduces to:

\[
f(i_L | 1 - \gamma) > f(i_L | 1).
\] (2.1)
2.1 Information and Auditing

The donor knows the realization $i_t$, but may choose to report another value $\hat{i}_t \in \{i_L, i_H\}$ to the recipient. The recipient cannot observe either the output level each period, $i_t$, or the investment level $I_t$, but may (at a cost $C > 0$) audit the donor. Auditing reveals the output level $\hat{i}_t$ for the current period to the recipient, but does not reveal the investment level $I_t$. In addition, auditing in this framework is stochastic–if the audit was deterministic, it has been shown that the optimal contract will be a standard debt contract. As we do not have enough information about actual contracts to conclude that a standard debt contract is justified, we allow for more complicated contracts by admitting the possibility that auditing is random.

The probability of auditing each period depends upon the reported output level $\hat{i}_t$ by the donor, i.e.

$$\Pr(\text{audit} | \hat{i}_t) := p(\hat{i}_t), \hat{i}_t \in \{i_L, i_H\}, p(\cdot) \in [0, 1]. \quad (2.2)$$

The investment level $I_t$ is never observable by the recipient. Thus, the recipient holds expectations by forming the probability $P_t$ that the donor has underinvested ($I_t = 1 - \gamma$) in period $t$. Expectations are updated in a Bayesian fashion to $P_{t+1}$ whenever the recipient audits the donor and hence learns the true level of output $i_t$. If there is no audit, however, then expectations remain unchanged at $P_t$:

$$P_{t+1}(I_t = 1 - \gamma | \hat{i}_t) = \begin{cases} \frac{f(\hat{i}_t | 1 - \gamma)P_t}{f(\hat{i}_t | 1 - \gamma)P_t + f(\hat{i}_t | 1)P_t(1 - P_t)} & \text{if audit} \\ P_t & \text{if no audit} \end{cases} \quad (2.3)$$

If the recipient chooses not to audit the donor, then there is a transfer between the donor and the recipient $\tau(\hat{i}_t, P_t)$ which depends upon both the reported level of output $\hat{i}_t$ and the current beliefs held by the recipient as to whether or not the donor has underinvested $P_t$. We think of such a transfer as either a tax or a rebate on the activities of the
investment project. If, on the other hand, the recipient chooses to audit the donor, then there is a ‘penalty’ which is assessed on the donor of $L(\hat{i}_t, i_t, P_{t+1})$, which depends upon the reported output level $\hat{i}_t$, the actual output level found from the audit $i_t$, and finally the updated expectations regarding whether or not the donor has underinvested $P_{t+1}$.

2.2 Preferences

For simplicity we assume that both parties are risk-neutral and that there is no discounting over the investment horizon $T$. In addition to its simplicity, such an assumption may be plausible in the case that both donor and recipient are (or represent) large institutions, which focus more upon return and less upon variations in the realized investment stream. We do, however, assume that the preferences of donor and recipient are somewhat aligned—in particular, they both prefer to have higher output levels than lower levels.

The rationale for this is twofold. First, the donor might have preferences for a higher volume of realized investment, as this may be measured as part of the “aid target” for the donor agency. Second, and perhaps more importantly, a model describing the investment project’s final benefit is incomplete without a precise treatment of how the local population will actually utilize the project upon completion. In our simple environment we do not model the local population at all—but we proxy the local population’s valuation of the project by including the level of realized output as an objective for both donor and recipient. Thus, valuing higher realized levels of output can be interpreted as the donor and recipient valuing, in an imprecise sense, the welfare of those who use the final project.\(^2\)

\(^2\)Naturally, a careful treatment of the local population is vital to select the correct criteria for assessing the success or failure of an investment project. We intend to model a decentralized local population in future research, by utilizing recent advances in agent-based computational economics. This extension is discussed briefly in Section 4 below.
place:

$$W^d := \max \{ I_s, i_s \} \sum_{t=1}^{T} \mathbb{E}_{f(i_t \mid I_t)} \left[ i_t + (1 - I_t) - p(\hat{i}_t) L(\hat{i}_t, i_t, P_{t+1}) - (1 - p(\hat{i}_t)) \tau(\hat{i}_t, P_t) \right]. \quad (2.4)$$

By contrast, the recipient selects auditing probabilities and transfers/penalties to maximize the sum of outputs and transfers net of auditing costs, beginning in period 1 with a prior probability $P_1$ that the donor underinvests:

$$W^r := \max_{\{ p, \tau, L \}} \sum_{t=1}^{T} \left\{ P_t \left( \mathbb{E}_{f(i_t \mid 1 - \gamma)} \left[ i_t + p(\hat{i}_t)(L(\hat{i}_t, i_t, P_{t+1}) - C) + (1 - p(\hat{i}_t)) \tau(\hat{i}_t, P_t) \right] \right) 
+ (1 - P_t) \left( \mathbb{E}_{f(i_t \mid 1)} \left[ i_t + p(\hat{i}_t)(L(\hat{i}_t, i_t, P_{t+1}) - C) + (1 - p(\hat{i}_t)) \tau(\hat{i}_t, P_t) \right] \right) \right\}. \quad (2.5)$$

It is worth noting once more that realized investment is a part of the return function for the aid recipient, reflecting the fact that (absent the more realistic modeling of the local population mentioned earlier and discussed below) the recipient has contracted with the donor to provide a service which benefits the recipient nation’s inhabitants.

The feasibility constraints on the transfers and penalties are:

$$\tau(i_j) \leq i_j, \ L(i_j, i_k, P) \leq i_k, \ j, k \in \{ L, H \}, \ \forall P. \quad (2.6)$$

Naturally, other feasibility constraints are possible if there are e.g. credit or liquidity constraints which might further modify the range of feasible transfers and penalties. In addition, it may be more realistic to consider the possibility that the donor can be fined for many periods following misreporting instead of just the current period. But the single period penalty adopted here is sufficient to motivate the idea that the donor is held accountable to the recipient in the event of misreporting.

It is true that in order for transfer and penalty payments to make sense, it must be possible for such payments to be enforceable if they should become necessary to impose.
This requires either 1) an institutional framework which has enforcement ability (such as a “World Court”, court of arbitration, or other third-party negotiating entity), or 2) a self-enforcing contract whereby the contracting parties shall in their own best interest agree to submit to transfers and penalties when they come to pass.

This latter case is quite interesting, as it is the only enforcement mechanism which can take place when institutions are weak. Such self-enforcement may take place if both donor and recipient are valuing not only the current project, but also the possibility of forming other development aid contracts with each other in the future. In this case, both have an incentive to act ‘as if’ institutions enforcing transfers and penalties exist, and we shall assume that they do so in what follows. The actual mechanism by which the donor and agent commit to self-enforcement, or commit to dispute resolution via a third party, is a subject of future research.

2.3 The Dynamic Programming Problem

As it stands, selecting an optimal contract for this framework would appear to be a daunting task, as the sets of optimal decisions resulting from the maximization problems (2.4) and (2.5) appear to be history dependent. Fortunately, the problem can be recast as a dynamic program using the approach of Monnet and Quintin (2005), which is based in turn upon Spear and Srivastava (1987). This approach uses a promised income level (or ‘continuation value’) as a state variable, which (along with the current state of expectations held by the recipient) summarizes all relevant information from the past.

As with transfers and penalties, there must also exist a mechanism to ensure that the agent (here the donor) believes the promised income can be a creditable contracting component. If the donor believes that the recipient in turn may expropriate future investment streams and not credibly commit to promised future income transfers, then the contract may not be implementable. We are currently examining both the United Nations
Treaty Collection and the World Bank Project Database, to examine how parties develop enforcement mechanisms for both transfer and penalty functions as well as continuation values for remaining in the project.

Each period the recipient optimizes the expected value of future net realized investment streams, \( \Pi_t \), by choosing an audit probability \( p \), a transfer value \( \tau \) in the event of no audit, a penalty value \( L \) in the event of an audit, and three promised expected income levels for the donor:

1. \( V^a(\hat{i}_t, i_t) \), which is the promised expected income level in the event of an audit, where the donor reports \( \hat{i}_t \) and the truth is revealed to be \( i_t \), and

2. \( V^n(\hat{i}_t) \), the promised expected income level when an audit is not performed. In this case, the promised income can only depend upon the donor’s report \( \hat{i}_t \).

As shown in Spear and Srivastava (1987), the expected income level, or ‘continuation value’, promised to the donor becomes a state variable. When choosing a vector \((p, \tau, V^a, V^n)\) in period \( t \), the recipient takes as given the promised income level from the previous period \( V \), and the current state of beliefs about whether or not the donor is underinvesting, \( P_t \).

This maximization problem is conditional upon a set of incentive compatibility conditions, which ensure that the donor chooses to report realized investment truthfully, and at the same time chooses to fully invest. In addition, the contract also defines a participation constraint: each period the donor is promised an expected income level \( V \), and this income level must be \textit{ex ante} attained, otherwise the donor chooses not to participate. In other words, the expected value of the current period’s income stream accruing to the donor, conditional upon truth-telling and full investment, must be equal to \( V \).

Finally, the recipient must also choose values for the promised income levels, transfers and penalties which are feasible. Although more complicated feasibility conditions are
possible, here we adopt relatively stringent constraints. First, the promised income levels cannot exceed the expected realized investment stream from the current period onward, in the event of audit and truth-telling, audit and lying, and no audit. Second, we assume without loss of generality that the project generates no realized investment stream after completion. And lastly, we suppose that the transfer and penalty amounts satisfy relations (2.6). As discussed earlier, this last assumption is particularly restrictive. It implies that any other assets held by the donor are unattainable by the recipient, and the only thing which is possible is to somehow ‘garnish’ the realized investment of the current period. The effects of more complicated punishment strategies, including termination of the project, multi-period seizure, and arbitration by a third party, are left for future research.

Formally, the dynamic programming problem looks like the following: each period $t$ the recipient seeks to find

$$\Pi_t(V, P_t) = \max_{\{p, \tau, L, V^n, V^a\}} \mathbb{E}_{f(i_t|1)} \left[ i_t + p(i_t)(L(i_t, i_t, P_{t+1}) - C + \Pi_{t+1}(V^a(i_t, i_t), P_{t+1})) + (1 - p(i_t)) (\tau(i_t, P_t) + \Pi_{t+1}(V^n(i_t), P_t)) \right]$$  \hspace{1cm} (2.7)

such the donor’s incentive compatibility conditions hold,

$$\mathbb{E}_{f(i_t|1)} \left[ i_t + p(i_t)(-L(i_t, i_t, P_{t+1}) + V^a(i_t, i_t)) + (1 - p(i_t))(-\tau(i_t, P_t) + V^n(i_t)) \right] \geq$$

$$\mathbb{E}_{f(i_t|1)} \left[ i_t + (1 - I_t) + p(\hat{i}_t)(-L(\hat{i}_t, i_t, P_{t+1}) + V^a(\hat{i}_t, i_t)) + (1 - p(\hat{i}_t))(-\tau(\hat{i}_t, P_t) + V^n(\hat{i}_t)) \right], \forall I_t, \hat{i}_t,$$ \hspace{1cm} (2.8)

the promised income is achieved,

$$V = \mathbb{E}_{f(i_t|1)} \left[ i_t + p(i_t)(-L(i_t, i_t, P_{t+1}) + V^a(i_t, i_t)) + (1 - p(i_t))(-\tau(i_t) + V^n(i_t)) \right], \hspace{1cm} (2.9)$$
the final value of the project is a constant (which may be taken to be zero),

$$\Pi_{T+1} = 0,$$  \hspace{1cm} (2.10)

the promised income values are feasible,

$$V \leq (T + 1 - t) \mathbb{E}_{P_t} \left[ \mathbb{E}_{f(i_t | I_t)} i_t \right],$$

$$V^n, V^n \leq (T - t) \mathbb{E}_{P_t} \left[ \mathbb{E}_{f(i_t | I_t)} i_t \right],$$  \hspace{1cm} (2.11)

and finally equations (2.6) hold.

Here, the notation $\mathbb{E}_{f(i_t | I_t)}$ denotes the expected value under the conditional probability distribution of realized investment $i_t$ given the investment level $I_t$ of the donor. Similarly, $\mathbb{E}_{P_t}$ is the expected value under the probability $P_t$ that the donor is underinvesting.

### 3 Computing the Model

The goals of this study are to construct a dynamic contracting framework for investigating development aid problems and also to solve such models based upon parameters which define the institutional and informational framework that such aid contracts operate within. One open challenge at this point is to better understand how complicated such a model is to solve in the face of development aid contracting data, and indeed how difficult it might be to solve such a model to find the optimal contract given a set of real-world parameters.

The original Monnet and Quintin (2005) model is analytically tractable—they find that optimal contracts will involve either total seizure of current output by the principal, or else the principal promises to the agent that no such seizures will occur in the future. In their framework there is no agent (here, donor) investment, and the only decision of the agent is whether or not to report output truthfully. In our model, by contrast, there
is unobservable investment and hence the recipient must introduce a new state variable, which is the belief held about whether or not the donor is underinvesting. (In addition, current work in progress aims to introduce both bilateral asymmetry and local population dynamics into the model, so that the model’s complexity will be increased even further.)

At this point, there is a choice to be made regarding the investment of time and resources into either a full analytical treatment of the simplified model, or into formulating a computational version of the model to be treated parametrically. We have opted for the latter approach not because it is necessarily superior to an analytical solution of a version of the model which can be solved, but rather because our expectation is that the full model with bilateral asymmetry and local population dynamics will preclude such techniques from being widely applicable. In this sense we shall put all our eggs into one basket and begin with computationally solving the model, demonstrating that in spite of its complexity one can draw meaningful conclusions by exploring how to state the problem in a computationally tractable fashion.

This is not to say, of course, that the model is trivially solvable computationally. Indeed, the benchmarking results we present below indicate that moving to the full model will require substantial computational resources both to solve a specific parametrization of the model (which would represent a particular development environment) and also to solve across sets of parameters (representing both a spectrum of development environments and also a form of robustness testing). But they do provide an initial starting point for our investigation into how the computational complexity of the model is related to the parameter set size, and also provides some information into how to most efficiently code the dynamic contracts to best take advantage of the available resources.
3.1 The Benchmark Computations

3.1.1 Monnet and Quintin (2005)

For the Monnet and Quintin (2005) model, the following parameters and ranges for the continuation value were adopted:

- \( i_H = 1, i_L = 0, C = 0.1, T = 2, \)
- \( f(i_L) = f(i_H) = 0.5, \)
- Continuation Value \( V: \) in period 2, \([0, 0.5]\). In period 1, \([0, 1]\).

The model was computed using multidimensional grid search, as this technique is both straightforward to implement and also amenable to scaling upwards for multiprocessor calculations. In the benchmark model, there are 15 variables which must each be partitioned into a grid: \( V, p(i_j), L(i_j, i_k), \tau(i_j), V^n(i_j, i_k), \) and \( V^n(i_j) \). We selected a grid size of 5 for each variable, resulting in \( 5^{15} \approx 30 \) billion grid points for evaluation. Each grid point required an evaluation of roughly 200 floating point operations, resulting in a naive estimate of ca. 6 trillion operations to be performed per time period. These operations were performed on the NCSA’s ‘cobalt’ cluster, which is an SGI Altix cluster of 1,024 Itanium2 processors. We utilized a single such processor, and the calculations took about 45 minutes to complete.

The results confirmed the analytical predictions of Monnet and Quintin (2005) for the optimal contracts to behave–either the principal seized all of the output realization for the current period, or promised a level of income to the recipient equal to that obtained when the principal did not engage in seizure in the future.
3.1.2 The model with Underinvestment and Expectations

The second benchmarking computation was performed using the illustrative model described in Section 2, which includes unobserved output, unobserved investment, and the expectations of the recipient about the donor’s investment.

The parameters and ranges used were:

- \( i_H = 1, i_L = 0, C = 0.1, T = 2, \)
- \( I_t \in \{1, 1 - \gamma\}, \gamma = 0.3, \)
- \( f(i_L|1) = 0.2, f(i_L|1 - \gamma) = 0.8, \)
- Continuation Value \( V \): range depends upon ex ante expected income in each period, which in turn depends upon expectations of recipient on underinvestment by donor—see equation (2.9).

Again, multidimensional grid search was performed—in addition to the variables listed in the first benchmarking exercise above, the expectation \( P_t \) held by the recipient about the donor’s investment level was also partitioned into a grid. Using again the standard of 5 grid points per partition, the total number of grid points considered increased by a factor of five over the first benchmark, to 150 billion points. In addition, due to the additional state variable given by promised income, the number of floating point operations per grid point increased to 300. Thus, for the second benchmarking exercise the total naive number of floating point operations per time period was around 45 trillion.

The computations were once more performed on the ‘cobalt’ SGI Altix cluster at the NCSA, and the resulting simulation ran on two Itanium2 processors, taking around an hour and a half. The resulting optimal contracts found confirm the intuition that expectations matter for enforcement—for those states in which the expectations of the recipient that the donor was underinvesting were low, the donor optimally chose to underinvest
and so the incentive compatibility conditions were violated. Above a certain level of ‘skepticism’, however, the expectations of underinvestment were high enough to deter underinvestment, and the incentive compatibility conditions were satisfied.

3.2 Summary

Thus, although for the original Monnet and Quintin (2005) model no computation is required as analytical solutions are obtainable, the benchmark computations replicate these solutions and show that the problem scales roughly as \(2 \times n^{15} \times 10^2\), where \(n\) is the number of grid points allocated to each variable, with around 6 trillion floating point operations. By contrast, the model with underinvestment the is more complicated both in terms of scaling and also in terms of operations. The problem scales as \(3 \times n^{16} \times 10^2\) (again \(n = 5\) in the benchmark calculations) but the number of floating point operations increases to around 45 trillion.

In addition, the model with underinvestment does not have a contract available for every state—in particular, there are states for which the recipient believes too strongly that the donor will not underinvest, causing one or more incentive compatibility conditions to be violated. This reflects the fact that a certain amount of ‘healthy skepticism’ is required on the part of the recipient to induce the donor to fully invest. If there is too much trust, i.e. the expectations of the recipient are high that the donor fully invests, then the donor will find its best interests served by always expropriating a portion of the investment stream.

Using these benchmarking solutions, doubling the number of grid points for the model with underinvestment to \(n = 10\) would necessitate roughly 65,000 times the processing time for the \(n = 5\) case. Using an SGI Altix cluster of 1,024 processors, for example, would reduce the problem to about 3.5 days of calculating to solve. Rather than take this back-of-the-envelope calculation as evidence of the “curse of dimensionality”, we find it
promising that a problem with something on the order of $65,000 \times 4.5$ trillion floating point operations is even discussable, much less solvable in less than a week. Clearly, with ‘intelligent’ grid selection and minimization of loops using modern techniques for numerical optimization these numbers could come down drastically, allowing even finer grids—and more complicated models—to be considered computable.

4 Bilaterally Asymmetric Information and Local Population Dynamics

The initial benchmarking results are encouraging. The simplified model outlined in the previous section is complex enough to capture the double uncertainty of donor investment and output reporting, yet is straightforward enough to enable us to compute the optimal contract using dynamic programming and relatively light computational resources. Unfortunately, the full model will require substantially more resources, and a simple accounting exercise shows that even the benchmark model above becomes daunting if the grid sizes are appreciably increased (again, an increase from 5 to 10 grid points necessitates $2^{16} \approx 65,000$ more grid points to consider). Thus, more sophisticated optimization routines and algorithms above and beyond brute force grid search would certainly have to be adopted and implemented.

The full model would be an extension of the previously introduced framework in two ways. First, there would be bilateral uncertainty: in addition to the donor’s unobserved investment level, the recipient may also engage in hidden activity. To capture this possibility we would modify the standard model above to allow the recipient to adjust the probability distribution of output, given the investment level of the donor. This captures the idea that through inefficiencies due to e.g. bureaucratic graft or undeveloped infrastructure, etc. the realization of output may itself be hampered by the recipient’s ‘type’.
Coupled with the hidden action problem of donor underinvestment, the full model would have donor and recipient each reflecting the roles of both principal and agent.

Second, and perhaps more importantly, it is not necessarily the case that the local recipient authority accurately represents the welfare benefit that the investment project is designed to attain. In particular, the local population—the population for whom the aid is intended—determines the final ‘return’ of the investment project. Both recipient and donor, then, must forecast the final utilization rate of the project by the local population in order to accurately assess the success of the project’s goals.

It is also not the case that the local population possesses either the technical acumen or information level necessary to engage in the type of formal contract accounting procedure outlined above. Rather, we feel that it is a more realistic assessment of a population’s behavior to introduce bounded rationality at the individual level. An individual agent (be they persons, households or other small aggregations) is influenced by the beliefs of its neighbors, and the problem of the population’s behavior becomes one more appropriately defined by agent-based computational economics and less by fully optimizing agents.

This observation, that the local population both matters and is complicated enough to model as a decentralized agent-based system, makes our argument to utilize high-performance computational resources even stronger. Lacking clear-cut analytical solutions in this case, it is imperative that a wide range of behavioral assumptions and parameter values be tested, so that a form of ‘sensitivity analysis’ can emerge. Contrary to such computational exercises as e.g. modern quantitative Macroeconomic analysis and ‘calibration theory’, it is likely that our augmented model will possess a rich set of nonlinear dynamics which renders the theory of perturbations around a unique stationary state difficult to justify. This is all the more reason to approach this type of model in exactly the same fashion as a computational physicist or chemist would approach a “many-body” problem, where the micro-dynamics are well understood and yet the problem exhibits
non-stationary or complex behavior that is more than the sum of its parts.

5 Conclusion and Future Research

In this paper we have introduced a general methodology to examine the problem of development aid investment. This methodology is presented using the language of dynamic contract theory, where the contract is between a donor authority and a local recipient of aid, and where the goal of the contract is to provide a stream of investment into a project with stochastic realized output.

In the most general (and realistic) environment, both the donor and the recipient must engage in some form of costly monitoring to ensure that the pre-commitments made regarding the final distribution and implementation of aid are honored. In addition, the valuation of the final benefit of donor aid must be described by the local population of final users, whose interests may not be aligned with the interests of either the donor authority or the local recipient authority who have engaged in the contract itself. As a first step toward such generality, we have treated here an extension of the Monnet and Quintin (2005) model where both investment and output are unobservable to the principal, and where the principal is the recipient of aid. Thus, we treat one half of the bilaterally asymmetric information case, a half which has until now been only given cursory examination in the literature. A fully bilateral model will, of course, have both donor and recipient each act as principal and agent, and the resulting contract must take into consideration the cynical-but-realistic view that self-interest can act against the stated objectives of each.

Our extension also uses per-period realized output as a proxy for local population valuation. Clearly this is untenable, and a more realistic model, as discussed earlier, is underway to model the local population. But even here we see that the objectives of
donor and recipient are not countervailing in every respect—by valuing realized output, for example, the donor must make an explicit trade-off between the marginal benefit of underinvesting and the marginal cost of a higher likelihood of low output. For the recipient the goal is to have an investment project of the highest ‘quality’ and so valuing realized output is natural—but in the case where the recipient could also choose to expropriate aid, this may too lead to a cost-benefit choice on the recipient’s part.

Overall, the computational results show that with access to high-performance computational resources it should be possible to develop and compute a full model of the flavor we have introduced here. This would allow us to, first, compare our results with existing contracts to see how well the computations match with or diverge from reality. In addition, it should then be possible to make policy suggestions for optimal contract design, helping to answer the question, “What ‘should’ aid contracts look like?” Roodman (2004) suggests that different nations have different levels of aid effectiveness, with the Scandinavian nations usually having a higher impact of aid per unit of aid delivered. Perhaps their aid contracts respect incentive compatibility conditions more than other nations, or perhaps another factor is responsible. We feel that our approach could be used to shed light on this issue.

Finally, we would like to hope that once the optimal contracting problem for development aid is better understood, it will be possible to design better contracts that can lead to more efficient allocations of aid, “and hence greater development”. This hoped-for improvement in aid efficiency would be beneficial in any event, but would be especially crucial in war and post-war environments (see e.g. Schiavo-Campo (2003), where the aforementioned difficulties become even more acute. A careful study of sustainable contracts for conflict areas is certainly of immediate interest.
References


