Value at Risk estimation for stock indices using the Basle Committee proposal from 1995

Momtchil Pojarliev and Wolfgang Polasek

Institute of Statistics and Econometrics
University of Basel
Holbeinstrasse 12
CH-4051 Basel
email: Momtchil.Pojarliev@unibas.ch
email: Wolfgang.Polasek@unibas.ch

August 4, 2000

Abstract

This paper compares different models for volatility forecasts with respect to the value at risk performance (VaR) for daily stock index returns. The VaR measures the potential loss of a portfolio for the next period at a given significance level. We will focus on the question if the choice of the appropriate volatility forecasting model is important for the VaR estimation. We compare the forecasting performance of several volatility models for the returns of the MSCI North America, the MSCI Europe and the MSCI Pacific indices. The resulting VaR estimators are evaluated with different criteria and with respect to the proposal of the Basle Committee from 1995. We propose an alternative calculation of the market risk charge (MRC).

Keywords: Volatility forecasts, Evaluation of VaR and market risk charges (MRC), Christoffersen tests.

1 Introduction

In the last years we witness a rapid development of the e-commerce and therefore also abnormal returns in internet stocks. Companies where the earnings per share (EPS) were negative had 1000% returns p.a. on the NASDAQ stock exchange in the beginning of 2000. Now, the time where the so called "dot-com" companies are rising rapidly and investors became millionaires over the night, seems to be over. In April 2000 the NASDAQ 100 index dropped by about 20%. Therefore the investors are interested in risk measures of their portfolios, like the VaR measure, the lower quantiles of the predicted returns. There are numerous approaches to calculate the VaR. Many analysts use the popular

variance-covariance approach by JP Morgan, better known as RiskMetrics [9]. Recently, new methods have been developed on the basis on extreme value theory [4]. The aim of the paper is to investigate the following question: Can time series forecasts improve the VaR estimates and what evaluation criteria for volatility forecasts are good diagnostics for the VaR performance? To answer the question we compare the performance of the following models: The naive model, where the variance estimator is just the historical variance, the RiskMetrics model, GARCH, t-GARCH, an asymmetric GARCH (AGARCH), power GARCH (PGARCH), exponential GARCH (EGARCH) and bivariate BEKK model. In a second step we evaluate the resulting VaR estimators.
The paper is organized as follows: In the next section we estimate the volatility of the MSCI North America, MSCI Europe and MSCI Pacific indices with the different univariate models. In section 3 we present and estimate the multivariate BEKK model. The forecasting performance of the different models is compared in section 4 using a rolling sample of 800 observation and an evaluation period of approximately two years. As evaluation criteria we use an auxiliary linear regression model for estimated and observed daily volatilities [11]. In section 5 we estimate the VaR of a hypothetical portfolio of 1 Mio $ which is invested in mutual funds and tracks the MSCI indices. We evaluate the VaR estimates using several criteria and with respect to the Basle Committee proposal from April 1995. We propose a new calculation of the market risk charge (MRC) which penalizes higher failure rates. In the last section we conclude.

2 Univariate volatility models

We will investigate the volatility of the daily returns of the MSCI North America, MSCI Europe and MSCI Pacific indices from May 1st 1995 until April 3rd 2000. The first 800 observations (from May 1st 1995 until May 22 1998) are used for the model selections and the rest for out-of-sample comparison. Figure 1 plots the labels and the returns of the MSCI North America index for the whole time horizon. Note the large day losses caused by the Asia crises at October 27 1997 (-6.65%) and by the Russian crises at August 31 1998 (-6.61%). In the following subsections we will describe the proposed models and discuss the estimation results for the MSCI North America index. The results for other two time series are just summarized.

2.1 The naive model

The naive model uses the variance of a moving sample of 800 observation (approximately 3 years) as forecast for the next period (1000 observations were used in Dockner and Scheicher, 1999), i. e.

$$\hat{\sigma}^2_{t+1} = \frac{1}{799}\sum_{i=1}^{800}(r_{t+1-i} - \hat{\mu})^2 \tag{1}$$

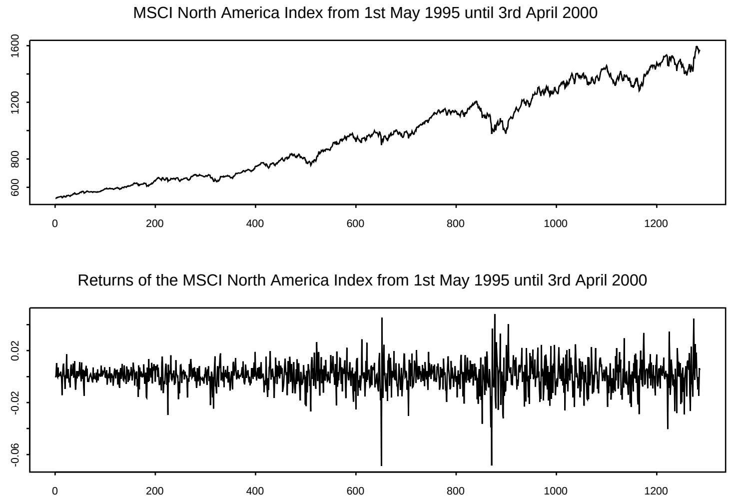


Figure 1: The MSCI North America index, labels and returns.

where r_t are the returns at time t and $\hat{\mu}$ is the estimated average return of the sample.

2.2 The RiskMetrics model

The model proposed by J. P. Morgan (1996) is an exponentially weighted moving average model. The volatility of the next period can be calculated as a weighted average of the current volatility and the squared return [9]

$$\sigma_{t+1}^2 = \lambda \sigma_t^2 + (1 - \lambda) r_t^2 \tag{2}$$

where λ is the weight factor. As proposed by RiskMetrics we set λ equal to 0.94. As an initial value for σ^2, we use the squared returns.

2.3 The GARCH model

We assume normally distributed returns

$$r_t | I_{t-1} \sim N[\mu_t, \sigma_t^2] \quad t = 1, ..., T, \tag{3}$$

where I_{t-1} is the information set up to time t or

$$r_t = \mu + \epsilon_t, \tag{4}$$

and the distribution of ϵ_t is assumed to be Gaussian with mean zero and variance σ_t^2. A GARCH(p,q) model for the parameterisation for the variance is as follow:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2. \tag{5}$$

The AIC and BIC values in Table 1 suggest to select a GARCH(1,1) model for the period from May 1st 1995 until May 22 1998 for the daily returns of the MSCI North America index. The estimated GARCH(1,1) model of the returns of the MSCI North America index is (t-values in parentheses):

$$\begin{array}{lcccccc} \hat{\sigma_t^2} & = & 10^{-6}1.23 & + & 0.083\epsilon_{t-1}^2 & + & 0.904\sigma_{t-1}^2 \\ (t-values) & & (2.48) & & (7.80) & & (57.47) \end{array} \tag{6}$$

The ACF of the squared returns of the MSCI North America index and the squared standardized residuals are shown in Figure 2. Notice the correlation in the squared returns which motivate the GARCH model. Since the ACF of the squared standardized residuals doesn't exhibit such a correlation structure we conclude that the GARCH (1,1) model works well. Nevertheless, we explore the fit of some alternative GARCH(1,1) models.

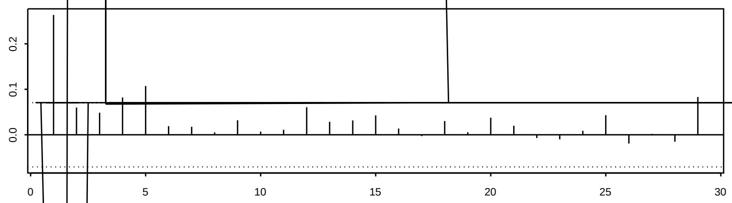
2.4 The GARCH model with t-distribution

Previous research has shown that the error term for returns models often has heavier tails than the Gaussian distribution. Therefore we have estimated a t-GARCH(1,1) model, where the t-distribution is used as conditional distribution for the residuals. The estimated model is similar to the normal GARCH(1,1) model in (6)

$$\begin{array}{lcccccc} \hat{\sigma}_t^2 & = & 10^{-7}7.07 & + & 0.041\epsilon_{t-1}^2 & + & 0.922\sigma_{t-1}^2 \\ (t-values) & & (1.73) & & (3.32) & & (43.29) \end{array} \tag{7}$$

	AIC	**BIC**
GARCH(1,0)	-4269	-4241
GARCH(1,0)	-5402	-5374
$GARCH(1,1)^*$	-5477	-5440
GARCH(2,1)	-5475	-5428
GARCH(2,2)	-5455	-5398

Table 1: AIC and BIC for different GARCH models, MSCI North America. The star (*) denotes the model with the smallest AIC and BIC values.



ACF of squared Observations

2.6 The Exponential GARCH model

An exponential GARCH model (EGARCH) has the following specification for the variance equation:

$$\ln \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \ln \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \ln \sigma_{t-i}^2 \tag{11}$$

The estimated variance equation for the returns of the MSCI North America index is:

$$\begin{array}{lcllllll} \ln \hat{\sigma}_t^2 & = & -0.271 & + & 0.100 \ln \epsilon_{t-1}^2 & + & 0.980 \ln \sigma_{t-1}^2 . \\ (t - values) & & (-3.30) & & (6.25) & & (130.10) \end{array} \tag{12}$$

2.7 Power GARCH model

The general power GARCH model (PGARCH) is defined as

$$\sigma_t^d = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^d + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^d \tag{13}$$

where the exponent d can be estimated or specified. We estimated the PGARCH(1,1) model as:

$$\begin{array}{lcllllll} \hat{\sigma}^d & = & 10^{-5}5.34 & + & 0.072 \epsilon_{t-1}^d & + & 0.934 \sigma_{t-1}^d \\ (t - values) & & (0.76) & & (7.11) & & (78.02) \end{array} \tag{14}$$

where $\hat{d} = 1.10$ (with a t-value of 4.67). Note that the estimated coefficient sum up to 1.

3 Multivariate Models

In this section we want to investigate to what extend multivariate models can improve the variance forecasts. Let $r_t = (r_{1t}, r_{2t})'$ be a vector of returns then a Gaussian multivariate GARCH model is given by

$$r_t | I_{t-1} \sim N[\mu_t, H_t] \quad , t = 1, ..., T. \tag{15}$$

where $\mu_t = \mu$ is the conditional mean and H_t the conditioanal covariance matrix given the information set I_t up to time t. We estimate two bivariate BEKK models, the first for the returns of the MSCI Europe and MSCI North America indices and the second for the returns of the MSCI Pacific and MSCI Europe indices.
The BEKK(p,q) model of Engle and Kroner (1995) assumes the following parameterisation of the conditional covariance matrix:

$$H_t = A_0 A_0' + \sum_{i=1}^{p} A_i (\epsilon_{t-i} \epsilon_{t-i}') A_i' + \sum_{i=1}^{q} B_i H_{t-i} B_i'. \tag{16}$$

Assuming the transposed matrix pairs for each of the coefficient matrices A_i and B_i guarantee symmetry and non-negative-definiteness of the conditional covariance matrix H_t. Using the AIC criteria, we select a BEKK(2,1) model for the returns of the MSCI Europe and MSCI North America indices and a BEKK(1,1) model for the returns of the MSCI Pacific and MSCI Europe indices. Table 2 shows the AIC and BIC values for different orders of the model.

	AIC	**BIC**
BEKK(1,0)	-11003	-10919
BEKK(1,1)	-11278	-11156^*
BEKK(2,1)	-11305^*	-11145
BEKK(2,2)	-11176	-10980

Table 2: AIC and BIC values for different BEKK models for MSCI Europe and MSCI North America indices. The * indicates the smallest value.

The ACF of the squared norm of the returns of the MSCI Europe and MSCI North America indices exhibits significant correlation which motivates multivariate GARCH model. In Figure 3 we plot the ACF of the squared norm of the standardized residuals for the fitted BEKK(2,1) model. Notice that all significant autocorrelation is removed. Figure 4 plots the estimated conditional standard deviations of the BEKK(2,1) model for the MSCI Europe and MSCI North America.

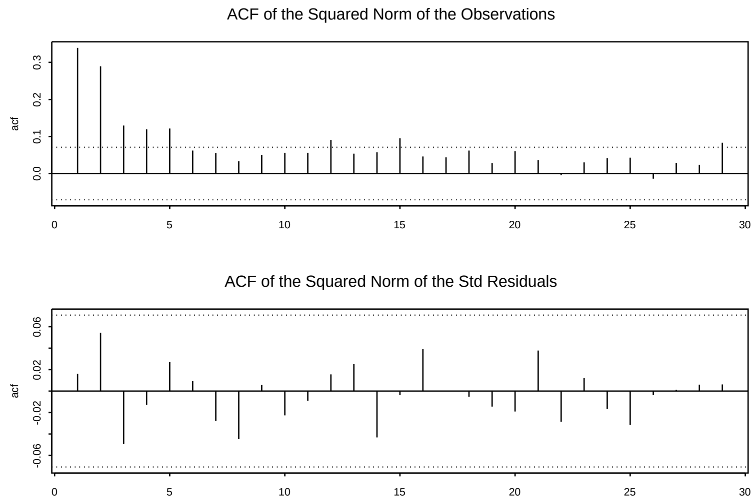


Figure 3: The ACF of the "squared norm returns" of the MSCI Europe and MSCI North America indices and the squared norm of the standardized residuals

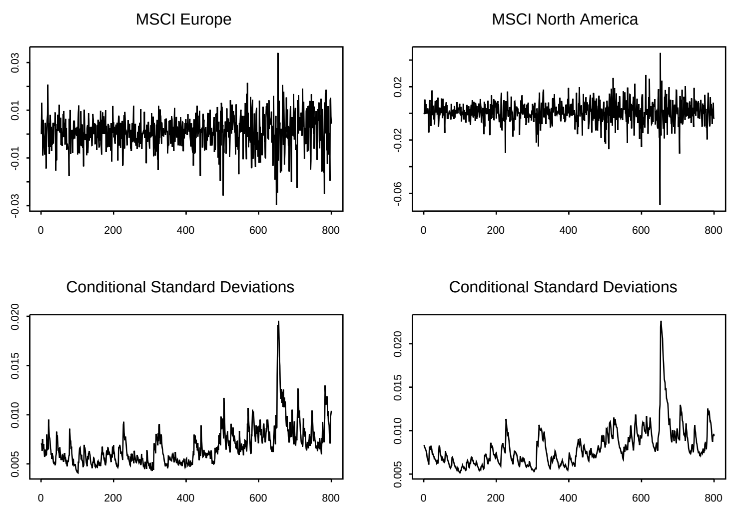


Figure 4: Returns and estimated conditional standard deviations of the BEKK(2,1) model for the MSCI Europe and the MSCI North America indices

4 Forecasting performance

Using the estimation results of the previous two sections and a rolling sample of 800 trading days w the results of the auxiliary regression (17) for different volatility models for the three regions (for 486 trading days and a rolling sample of 800 observations).

From tables 3, 4 and 5 we see that the naive model performs much worse than the time series models. The BEKK model and the asymmetric GARCH lead to a larger R^2 than the other models for all three indices.

There are significant differences in the predictive performance for the three regions. The volatility of the MSCI Europe index is better to forecast than the volatility of the MSCI Pacific index or the MSCI North America index for the period 1998-2000. However, the choice of the squared returns as proxy for the actual volatility is questionable and the resulting R^2 should be interpreted with care.

In the next section we investigate the effect of the different volatility models on the value at risk estimation.

	α(t-st.)	β(t-st.)	R^2
Naive	0(3.34)	-1.13(-1.47)	0.004
RiskMetrics	0(2.46)	0.575(3.52)	0.025
GARCH(1,1)	0(2.83)	0.571(3.98)	0.032
t-GARCH(1,1)	0(2.16)	1.026(3.97)	0.031
AGARCH(1,1)	0(3.45)	0.593(5.45)	0.058
EGARCH(1,1)	0(2.02)	0.677(3.19)	0.021
PGARCH(1,1),$p = 1.11$	0(2.49)	0.568(3.15)	0.020
BEKK(2,1)	0(0.13)	1.034(5.83)	0.062

Table 3: Model comparison by auxiliary regression of the MSCI North America index (1998/05/25 - 2000/04/03).

	α(t-st.)	β(t-st.)	R^2
Naive	0(6.66)	-2.27(-4.05)	0.030
RiskMetrics	0(1.76)	0.811(7.58)	0.106
GARCH(1,1)	0(1.61)	0.812(7.77)	0.111
t-GARCH(1,1)	0(1.75)	0.876(7.31)	0.110
AGARCH(1,1)	0(1.47)	0.869(8.36)	0.126
EGARCH(1,1)	0(1.84)	0.754(7.57)	0.106
PGARCH(1,1),$p = 1.11$	0(1.84)	0.852(7.42)	0.102
BEKK(2,1)	0(1.85)	0.759(7.63)	0.108

Table 4: Model comparison by auxiliary regression of the MSCI Europe index (1998/05/25 - 2000/04/03).

	α(t-st.)	β(t-st.)	R^2
Naive	0(4.62)	-3.139(-3.35)	0.022
RiskMetrics	0(2.38)	0.516(2.93)	0.017
GARCH(1,1)	0(2.43)	0.478(2.95)	0.017
t-GARCH(1,1)	0(1.90)	0.779(3.09)	0.019
AGARCH(1,1)	0(0.60)	0.939(3.72)	0.028
EGARCH(1,1)	0(1.42)	0.653(2.85)	0.016
PGARCH(1,1),$p = 1.11$	0(1.44)	0.626(2.99)	0.018
BEKK(1,1)	0(1.60)	0.578(3.95)	0.031

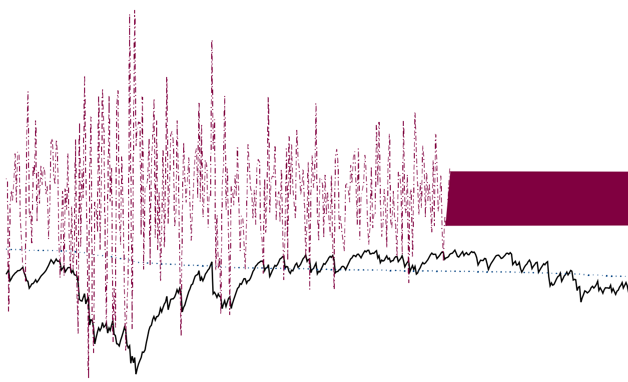
Table 5: Model comparison by auxiliary regression of the MSCI Pacific index (1998/05/25 - 2000/04/03).

5 VaR comparison

We assume a hypothetical portfolio of 1 Mio US \$ which consists of the MSCI North America, MSCI Europe and MSCI Pacific indices and the VaR is estimated for the next trading day (in the period 25 May 1998 - 3rd April 2000). Assuming that the returns are normally distributed, the 95%-VaR is computed as 95% quantile of the returns distribution i.e.

$$\widehat{VaR}_{t+1} = -10^6 1.65 \hat{\sigma}_{t+1} \tag{18}$$

where $\hat{\sigma}_{t+1}$ is the forecast of the standard deviation given all information up to time t. A 95%-VaR of 1000 US \$ means that with 5% probability the loss for the next trading day will be more than 1000 US \$. Figure 6 plots the actual portfolio changes ($10^6 r_t$) when tracking the MSCI Europe index, the VaR estimates based on the GARCH(1,1) model and the VaR estimates based on the naive model. Note that the naive model doesn't capture the risk for the first 200 observations and overestimates it in the rest of the period. Using the VaR estimates of the naive model would overestimate the risk if the volatility is small and underestimate it in bear markets.



5.1 Evaluating the VaR by interval forecasts

To evaluate the performance of the different 95%-VaR estimators we are using the following four criteria:

(a) The failure rate ($\bar{F}$)

(b) Likelihood ratio test of unconditional coverage (LR_{uc})

(c) Likelihood ratio test of independence (LR_{ind})

(d) Joint test of coverage and independence (LR_{cc})

We define the number of failures (F) as the number of times for which the actual loss is larger than the estimated VaR. For $t = 1, ..., T$ we define the failure rate to be the sample average

$$\bar{F} = \frac{1}{T}\sum_{t=1}^{T} D_t \tag{19}$$

with the dummy variable

$$D_t = \begin{cases} 1 & \text{if} \quad VaR_t - P_t^a > 0; \\ 0 & \text{if} \quad -VaR_t - P_t^a \leq 0. \end{cases} \tag{20}$$

The following LR_{uc}, LR_{ind} and LR_{cc} tests are proposed by Christoffersen (1998) to evaluate forecasts over a certain horizon. The likelihood ratio (LR) test of unconditional coverage LR_{uc} tests if $E(D_t) =$ against $E(D_t) \neq$ where α is the probability level for the VaR and T is the number of trading days ($T = 486$) in the evaluation period.
The LR test of independence LR_{ind} tests the hypothesis of independence against a first order Markov chain. Independence would mean that the days for which the actual losses are larger than the estimated value-at-risk ($D_t = 1$) are independent from each other.
The above tests are combined into LR_{cc} test, where the null of the unconditional coverage test is tested against the alternative of the independence tests. The three tests are numerically related by the following equation (see Christoffersen, 1998):

$$LR_{cc} = LR_{uc} + LR_{ind}. \tag{21}$$

The results of the Christoffersen tests for the estimated 95%-VaR based on the BEKK(2,1) model of the MSCI North America index are presented in Figure 8. The three lines in each panel indicates the values of the LR statistics from 99%-VaR until 90%-VaR. The horizontal line corresponds to the 5 per cent critical value of the relevant chi-squared distribution. We can see that the null hypotheses are not rejected at α-levels between 4% and 10%. Since the LR_{cc} and LR_{uc} tests are rejected for 99%-VaR we conclude that the BEKK model might not be the optimal model for a level of $\alpha = 1\%$.

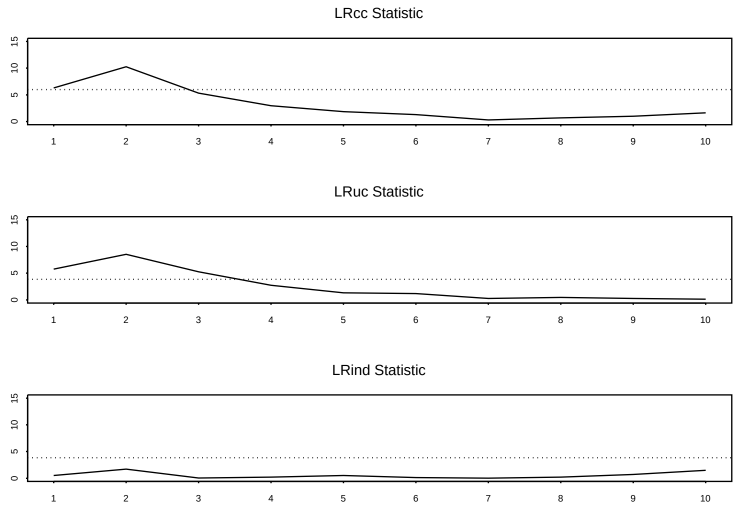


Figure 7: Christoffersen tests of the $(1-\alpha)\%$-VaR (for α between 1% and 10%) for the BEKK(2,1) model of the MSCI North America index

Tables 6, 7 and 8 summarize the result for the out-of-the-sample performance of the VaR models for the different evaluation criteria. The + and - for the Christoffersen tests mean that the null hypothesis is accepted or rejected at a α level of 5% respectively. We can see that although according to the auxiliary regression approach of Pagan and Schwert (1990) the volatility performance for the returns of the MSCI Pacific index is the worst, the 95%-VaR estimates are good for all volatility models. It is surprising that poor volatility forecasts (the highest R^2 for the MSCI Pacific index was 0.031) can lead to good VaR estimates. An explanation could be that the MSCI Pacific index has increased by 47% between May 25 1998 and April 3rd 2000. For comparison: The MSCI North America index rose 32% and the MSCI Europe index 10%.
There is a clear tendency that the multivariate volatility models are better for volatility forecasts and VaR estimates. Note that all BEKK models lead to better VaR estimates. Also the empirical failure rate of the 95%-VaR based on the BEKK model is the lowest one.

5.2 The Basle Committee proposal from 1995

In April 1995, the Basle Committe proposed an "internal model" approach which allows banks to use their own risk measurement models. An institution with significant trading activity has to provide a capital charge for the market risk using either its own internal risk measurement model ("the internal model approach") or a "standardized" VaR model developed by the Basle Commit-

	LR_{uc}	LR_{ind}	LR_{cc}	F ($\bar{F}$ in %)
Naive	-	+	-	39 (8.02)
RiskMetrics	+	+	+	31 (6.38)
GARCH	+	+	+	34 (6.99)
t-GARCH	-	+	-	37 (7.61)
AGARCH	-	+	+	35 (7.20)
EGARCH	-	+	+	36 (7.40)
PGARCH	+	+	+	33 (6.79)
BEKK	+	+	+	30 (6.17)

Table 6: 95%-VaR performance for 486 days, MSCI North America index.

	LR_{uc}	LR_{ind}	LR_{cc}	F ($\bar{F}$ in %)
Naive	-	-	-	45 (9.25)
RiskMetrics	+	+	+	34 (6.99)
GARCH	+	+	+	31 (6.37)
t-GARCH	+	+	+	32 (6.58)
AGARCH	+	+	+	32 (6.58)
EGARCH	+	+	+	32 (6.58)
PGARCH	+	+	+	30 (6.17)
BEKK	+	+	+	30 (6.17)

Table 7: 95%-VaR performance for 486 days, MSCI Europe index.

	LR_{uc}	LR_{ind}	LR_{cc}	F ($\bar{F}$ in %)
Naive	+	+	+	28 (5.76)
RiskMetrics	+	+	+	25 (5.14)
GARCH	+	+	+	23 (4.73)
t-GARCH	+	+	+	23 (4.73)
AGARCH	+	+	+	26 (5.35)
EGARCH	+	+	+	21 (4.32)
PGARCH	+	+	+	19 (3.90)
BEKK	+	+	+	17 (3.49)

Table 8: 95%-VaR performance for 486 days, MSCI Pacific index.

tee. The market risk charge (MRC) is calculated on the basis of a 99%-VaR estimates, calculated for a 10-day holding period. A supervisory multiplier k (a number of 3 or higher) is used to penalize failure rates of VaR in the last year. The MRC is calculated as the higher of (a) the previous day's 99%-VaR measure, calculated for ten days holding period, or (b) the average of the VaR measures during the preceding sixty business days, times the multiplier (k). The multiplier is determined by the number of exceeding (failures) in the last 250 business days. Thus the MRC at period t is given by

$$MRC_t = max(k\frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}^{MRC}, VaR_{t-1}^{MRC}) \tag{22}$$

where the $VaR_t^{MRC} = -10^{1/2}2.32\hat{\sigma}_{t+1}$ is the 99%-VaR for a 10 days holding period and 2.32 is the 99% quantile of the normal distribution. Table 9 shows the increase of the multiplier (k) in dependence of the failures proposed by the Bundesaufsichtsamt fuer Kreditwesen (BaKred) in Germany (see Zuccini and Neumann, 1999). For the evaluation of the VaR with respect to the MRC we

failures	increase in k
0-4	0.00
5	0.40
6	0.50
7	0.65
8	0.75
9	0.85
> 10	1

Table 9: BaKred specifications for the increase in the multiplier as a function of the failures.

use the following 4 volatility models: the naive model, the GARCH model, the AGARCH model (because of the large R^2 achieved by the auxiliary regression) and the BEKK model.
Tables 10, 11 and 12 show in the first column the average MRC in percentage of the portfolio of 1 Mio US $ for 426 business days time horizon. The failure rate is calculated for 250 trading days starting at May 25 1998. Thus the optimal failures are 2.5 days for the 99%-VaR. Note that the penalty function by the Basel Committee on Banking Supervision (MRC) can't distinguish between higher failure rates. Therefore we propose an alternative calculation of the MRC, where the minimum value of the multiplier k^* is 2 and increases linearly with the failures:

$$MRC_t^* = max(k^*\frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}^{MRC}, VaR_{t-1}^{MRC}) \tag{23}$$

with

$$k^* = 2 + \frac{1}{2} max(0, failure \quad rate - 2.5) \tag{24}$$

The new penalty function leads to very high MRC for larger failure rates and shows that the naive model has a bigger distance to the other volatility models. On the other side the MRC^* is much smaller than the MRC for models with lower failure rates. Note the larger spread of the new MRC^* values for the 3 indices. For each index region a different volatility model is selected if according the MRC or MRC^* measures are minimized. The BEKK model is the best for the North America, the RiskMetrics model for Europe and the AGARCH model for the Pacific index.

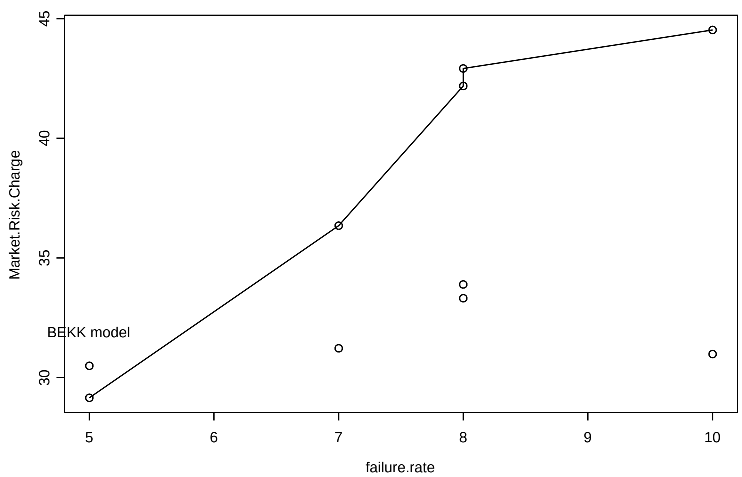


Figure 8: The average market risk charge (MRC and MRC^*) for different failures of the MSCI North America index. The line connects the values of the new market risk charge (MRC^*) which is a monotone function of the failures

It is surprising to see that the MRC in (22) which is based on the Basle comittee and the multiplier k as proposed by the BaKred leads to nonlinear behaviour when it is evaluated for different failure rates. In table 10, for instance, BEKK has a 2% failure rate and the naive model has a 4% failure rate. The MRC for the BEKK model is 30.49% but for the naive model is 30.98% while the RiskMetrics model with 3.2% failure rate has a 33.89% MRC value. This nonlinear dependence is not possible for the MRC^* values which depend linearly on the failure rate (in the range 29.15 to 44.53). For all three indices the MRC^* is smaller than the MRC if the BEKK model is used to forecast the VaR. This shows that the opportunity costs of a bank can be reduced if a good VaR model is used.

	MRC (in %)	MRC^*	F ($\bar{F}$ in %)
Naive	30.97	44.53	10 (4.0)
RiskMetrics	33.99	42.92	8 (3.2)
GARCH	33.30	42.19	8 (3.2)
AGARCH	31.21	36.35	7 (2.8)
BEKK**	30.49	29.15	5 (2.0)

Table 10: Average MRC for 426 days, failure rate is based on 99%-VaR for 250 days starting on May 25 1998, MSCI North America index. The stars (**) denote the model with the smallest MRC and MRC^* values.

	MRC (in %)	MRC^*	F ($\bar{F}$ in %)
Naive	27.91	78.50	21 (8.4)
RiskMetrics**	24.37	22.34	4 (1.6)
GARCH	27.83	26.60	5 (2.0)
AGARCH	27.61	26.39	5 (2.0)
BEKK	28.60	27.33	5 (2.0)

Table 11: Average MRC for 426 days, failure rate is based on 99%-VaR for 250 days starting on May 25 1998, MSCI Europe index. The stars (**) denote the model with the smallest MRC and MRC^* values.

	MRC (in %)	MRC^{**}	F ($\bar{F}$ in %)
Naive	38.49	48.76	8 (3.2)
RiskMetrics	32.90	30.16	4 (1.6)
GARCH	33.76	22.50	1 (0.4)
AGARCH*	31.54	21.03	2 (0.8)
BEKK	35.30	23.52	0 (0.0)

Table 12: Average MRC for 426 days, failure rate is based on 99%-VaR for 250 days starting on May 25 1998, MSCI Pacific index. The stars (**) denote the model with the smallest MRC and MRC^* values.

6 Conclusions

We have compared the performance of VaR estimators based on 6 univariate volatility models together with the historical variance and a multivariate BEKK model. The BEKK model gives the best volatility forecasts and leads to the lowest failure rate for a 95%-VaR. The R^2 of the auxiliary regression model can be used as indicator for the goodness of the resulting VaR estimates even if the R^2 values are very small. The Christoffersen tests are useful as diagnostic tests for a good VaR model when the α-level is very small. For the 99%-VaR model we have used the Basle Committee proposal from 1995 as evaluation criterion. An important result is that the used MRC measure doesn't discriminate enough for higher failure rates. Therefore, we propose a new calculation of the multiplier which leads to higher market risk charge for large failure rates and to lower market risk ccatorthef[[[,di[,sedre"q]:w,est"q]4BBBMB,failu4"q4[[[isilu4"q4[[[sry 9H,s"][[mall.

[10] Basel Committee on Banking Supervision, *An internal model-based approach to market risk capital requirements*, World Wide Web, `http://www.bis.org./publ/index.htm`, 1995.

[11] A. Pagan and W. Schwert, *Alternative models for conditional stock volatility*, Journal of Econometrics (1990), no. 50, 267–90.

[12] W. Zuccini and K. Neumann, *A comparison of several time series models for assessing the value at risk of shares.*, to appear in Applied stochastic models for busines and industry (ASMBI), 2000.