Portfolio construction using multivariate time series forecasts

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Abstract

This paper provides an analysis of how the forecasts of the returns of stock indices and their variance can e used for portfolio construction. We use a multivariate VAR-GARCH model to predict the monthly returns and the variance matrix of the MSCI North America, MSCI Europe and MSCI Pacific indices from Fe ruary 1990 until Septem er 1999.
We want to concentrate on the following research questions: first, how can forecasts of time series models e used for portfolio weight selection? Second, what kind of information improves portfolio performance? We compare two minimum-variance portfolios, a glo al portfolio ased only on the forecasted variance matrix, and the second portfolio ased on the forecasted returns and variance matrix. A comparison, ased on several criteria etween the portfolios and the enchmark shows that time series forecasts can e useful for active portfolio management. We have chosen the returns of the MSCI World index in US $ as a enchmark.

Keywords: Multivariate ARCH models, portfolio selection.

1 Introduction

The financial markets in the 1980's were dominated by the view: stocks returns are unpredictable, like coin flips. Therefore this view is the random walk hypothesis. What does this mean for the investors? Active portfolio management in this case will do nothing to improve one's portfolio over the long run: on the contrary, it simply increases transactions cost. Gains and losses in the short run were treated like a random walk. Portfolio managers were not thought to be able to beat market indices which lead to passive strategies of following the

market in the sense of minimizing the tracking errors.
Contrary to the view that stock market are efficient, previous studies have shown that the returns of assets are predictable (Harvey, 1991). Furthermore, the volatilities of stock returns can exhibit a rich interaction pattern, as was shown in Polasek and Ren (1999). We will show in section 4 that time series forecasts can improve portfolio performance.
Markowitz (1959) mean-variance efficiency is the classic paradigm of the modern finance theory for asset allocation. The Markowitz efficient frontier represents all efficient portfolios in the sense that all other portfolios have lower expected returns for a given level of risk (measured by the standard deviation). Using a classical mean-variance framework we investigate the extent to which optimal holdings for the three regions (North America, Europe and Pacific) depart from the benchmark weights. We estimate the optimal weights for the MSCI North America, MSCI Europe and MSCI Pacific with two different strategies. Portfolio one will be constructed using the one step ahead forecasted variance matrix of the monthly returns. For the estimation of the optimal weights of portfolio two we will use the one step ahead forecast of the first two moments of the returns distribution. A comparison between the two portfolios shows what additional information can improve the portfolio performance.
The paper is organized as follows: First we estimate a multivariate VAR-GARCH model with exogenous variables for the returns vector of the MSCI indices for the three regions. Section three describes a classical mean-variance framework and the methodology of constructing the two portfolios. In section four we compare the portfolios with the benchmark using common and additional criteria. In the last section we summarize our findings.

2 Multivariate time series forecasts

We forecast future returns μ and the covariance matrix $\mathbf{H}$ by a multivariate time series model. We found as best model a VAR(1)-X(0)-GARCH(1,1) model for the 3-dimensional returns vector (r^A, r^E, r^P) of the MSCI North America, MSCI Europe and MSCI Pacific indices using the AIC and BIC criteria. This can be seen from table 1, which shows the AIC and BIC for six different models.

	AIC	**BIC**
AR(1)-X(0)-GARCH(1,1)	2009	2208
AR(1)-X(0)-GARCH(1,2)	2015	2246
AR(1)-X(1)-GARCH(1,2)	2056	2354
AR(1)-X(1)-GARCH(2,1)	2070	2367
AR(1)-X(1)-GARCH(2,2)	2105	2436
AR(2)-X(1)-GARCH(2,2)	2136	2483

Table 1: AIC and BIC for different GARCH models

The notation VAR(1)-X(0) stands for a VAR model with exogenous variables with $lag = 0$. Table 2 gives a list of the exogenous variables which were used for model estimation. The time horizon is from February 1990 until September 1999. We are using monthly data.

Symbol	**Description**
B3m	Returns on the 3-month treasury bills rate
B10y	Returns on the US benchmark 10 years bond index
P/E	Returns on the price-earnings ratio for the US market
Yen/$	Returns of the exchange rate Yen to US $

Table 2: Exogenous variables of the VARX model

The stationarity of the returns was checked by classical unit root tests and the Bayesian method as in Polasek and Ren (1997). To reduce the number of parameters we specify a diagonal VAR(1)-GARCH(1,1)model with exogenous variables for the return vectors

$$\begin{pmatrix} r^A \\ r^E \\ r^P \end{pmatrix} = \begin{pmatrix} \mu^A \\ \mu^E \\ \mu^P \end{pmatrix} + \begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix} \begin{pmatrix} r^A_{-1} \\ r^E_{-1} \\ r^P_{-1} \end{pmatrix} + $$
$$+ \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} \epsilon_A \\ \epsilon_E \\ \epsilon_P \end{pmatrix}. \tag{1}$$

The variances are modeled by diagonal-vec model of order (1,1), where the conditional covariance matrix is given by

$$vec\mathbf{H}_t = a + diag(\mathbf{\Psi})\epsilon_t^2 + diag(\mathbf{\Phi})vec\mathbf{H}_{t-1}. \tag{2}$$

The constant a is positive and the matrices $\mathbf{\Psi}$ and $\mathbf{\Phi}$ are restricted to insure that the conditional covariance matrix $\mathbf{H_t}$ is positive definite and symmetric. For the estimation of the model we use the GARCH module of S-Plus (1996). The return vector and the exogenous variables are multiplied by 100 to get convenient estimation of the coefficients. The estimated mean equation are as follows (standard deviations in parenthesis):

$$\begin{array}{rcllll} \hat{r^A} & = & 1.212^* & - & 0.312^* r^A_{-1} & + & 0.041 B3m \\ & & (0.318) & & (0.104) & & (0.0767) \\ & + & 0.076 B10y & + & 0.448 P/E & - & 0.001 Yen/\$ \\ & & (0.1107) & & (0.126) & & (0.487) \end{array}$$

$$\begin{array}{rcllll} \hat{r^E} & = & 0.852^* & - & 0.101 r^E_{-1} & + & 0.345^* B3m \\ & & (0.362) & & (0.124) & & (0.203) \\ & + & 0.031^* B10y & - & 0.1588 P/E & + & 0.316^* Yen/\$ \\ & & (0.102) & & (0.163) & & (0.250) \end{array} \tag{3}$$

$$\begin{array}{rlllll} \hat{r^P} & = & -0.455 & - & 0.028 r^P_{-1} & + & 0.606^* B3m \\ & & (0.663) & & (0.087) & & (0.106) \\ & + & 0.316^* B10y & - & 0.134^* P/E & - & 0.731^* Yen/\$ \\ & & (0.292) & & (0.233) & & (0.235) \end{array}$$

The estimated parameters with a t-value greater than 1 are marked with a star (*). We can see that although we use as exogenous variables only economic variables from the USA, they are not significant for the US market (USA has a 97% weight in the MSCI North America index), but influence the return of the MSCI Europe and the MSCI Pacific indices. Increases in the Treasury bill 3 month rate and the yields of the 10 year bond index in the USA market have positive influence on the returns of the other two regions, which shows the high mobility of investment. Increasing rates in the USA will lead investor to reinvest in Europe and the Pacific. We can observe "hangover" of the Dow Jones index in February 2000 and in the same time increases in the DAX index, which means that the model estimations still holds out-of-the sample. Devaluation of the Yen/US $ exchange rate has a negative influence on the returns of the MSCI Pacific index and a positive influence on the returns of the MSCI Europe index. The estimation result confirms previous research results that the US market influences all other markets .

The forecasted variance matrix **H** (multiplied by 10000) of the returns of the MSCI North America, MSCI Europe and MSCI Pacific indices for October 1999 is as follows (correlation coefficients are in parenthesis)

MSCI N.America	MSCI Europe	MSCI Pacific
8.31	%	%
4.84(0.57)	8.62	%
5.99(0.41)	7.33(0.49)	26.08

3 Portfolio construction

Investors prefer portfolios with larger mean returns and lower risk (measured by the standard deviation) and they will accept more risk only if they get higher returns as compensation. This means that all investors should hold portfolios on the mean variance frontier. Any portfolio return on the frontier can be constructed as a combination between the market portfolio and the risk-free interest rate. The weighting of these two funds depends on the utility functions of investors. An investor who is extremely risk averse will invest only in the money market to get the risk free rate. In our empirical work we will investigate if the benchmark weights, the proportion of MSCI North America, MSCI Europe and MSCI Pacific indices in the MSCI World index are the optimal holdings and how an investor performs with active weights.

If $\mathbf{w}$ is the vector of the holding, $\boldsymbol{\mu}$ the vector of the expected returns of the assets and $\mathbf{H}$ the variance matrix of the returns, then the portfolio variance is $\sigma_p^2 = \mathbf{w}'\mathbf{H}\mathbf{w}$ and the portfolio return is $\mu_p = \mathbf{w}'\mu$. The optimization problem of a mean variance portfolio in absence of a riskfree asset is given by (see Campbell et al.1997):

$$min \mathbf{w}'\mathbf{H}\mathbf{w}$$

subject to

$$\mathbf{w}'\mu = \mu_p \quad and \quad \mathbf{w}'\iota = 1,$$

where ι is a vector of ones. The solution of this problem is:

$$\mathbf{w}_p = \mathbf{g}{+}\mathbf{h}\mu_p, \tag{4}$$

where $\mathbf{g}$ and $\mathbf{h}$ are $(N \times 1)$ vectors,

$$\mathbf{g} = \frac{1}{D}\left[B(\mathbf{H^{-1}}\iota) - A(\mathbf{H^{-1}}\mu)\right]$$
$$\mathbf{h} = \frac{1}{D}\left[C(\mathbf{H^{-1}}\mu) - A(\mathbf{H^{-1}}\iota)\right]$$

and $A = \iota'\mathbf{H^{-1}}\mu$, $C = \iota'\mathbf{H^{-1}}\iota$, and $D = BC - A^2$. A portfolio is called the *global minimum-variance portfolio* (GMV), if the weight vector is computed by the simpler formula

$$\mathbf{w}_{GMV} = \frac{1}{C}\mathbf{H}^{-1}\iota. \tag{5}$$

We will construct two different portfolios using the one step ahead forecast of the variance matrix in equation (2) and the forecast of the returns vector in equation (3).

Portfolio 1 is the GMV portfolio and based only on the forecasted variance matrix. We compute the portfolio weights for the in-sample period from February 1995 until September 1999 using the formula (5).

The weights of portfolio 2 are computed with the formula in equation (4). The required inputs are: the variance matrix $\mathbf{H}$, the vector of the expected asset returns μ and the portfolio return μ_p. We use the forecasted variance matrix $\mathbf{H_{t+1}}$ as for portfolio 1, but μ is computed as a 12 month moving average of the forecasted returns vector from (3). As portfolio return μ_p we choose 12% per year. To avoid negative weights we use the restriction $min\mathbf{w_i} = \mathbf{0.05}$.

Figure 1 shows the weights of the two portfolios for the in-sample period. We see that in the GMV portfolio the MSCI Pacific index has lower weights than in the benchmark for the first nine months of 1999 (The average benchmark weights for this period are 54%, 35% and 11%, respectively, for North America, Europe and Pacific). The reason is that the MSCI Pacific index was more volatile in 1999 than the other two series.

Portfolio 2, on the other hand, shows decreasing holdings in Europe and increasing holdings in the Pacific for the first nine months of 1999.

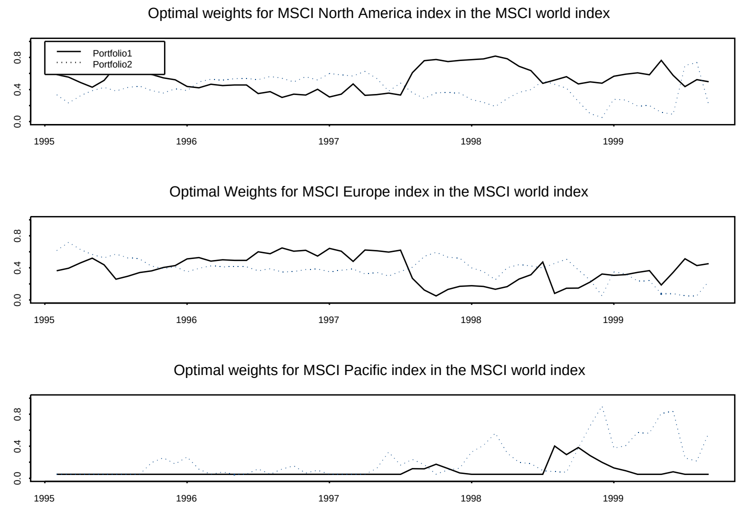

Figure 1: Portfolio weights for the in-sample period from February 1995 until September 1999

4 Comparison

We perform the back-testing with the estimated weights by comparing portfolio 1 and portfolio 2 with the MSCI World index by using the following criteria:

(a) mean return per year (in percent)

(b) standard deviation per year (in percent)

(c) cumulative return for 56 months, 3 years and year to date (in percent)

(d) Sharpe ratio

(e) success rate

The *Sharpe* ratio is defined as the expected excess return of portfolio P divided by the risk of portfolio P:

$$S_p = \frac{r_p - r_{riskfree}}{\sigma_p} = \frac{r_{excess}}{\sigma_p},$$

The risk free rate $r_{riskfree}$ is assumed to be 3% per year. The correlation between the given portfolio and the market portfolio is the ratio of Sharpe measures.

We define the *success rate* as the percentage of times (in months) in which the portfolio returns beat the benchmark returns. A portfolio based on active strategies should have a success rate higher than 50% in the long run.

Table 3 gives the summary of the comparison. Over 5 years portfolio 1 and 2 perform equally well. Portfolio 1 (based on global mean-variance weights) performs less well in 1999 than portfolio 2 which takes into account the forecasted portfolio returns.

Figure 2 shows the cumulative returns of the portfolios and the benchmark. We can see that both time series based portfolios yield about 14% more returns over 56 months.

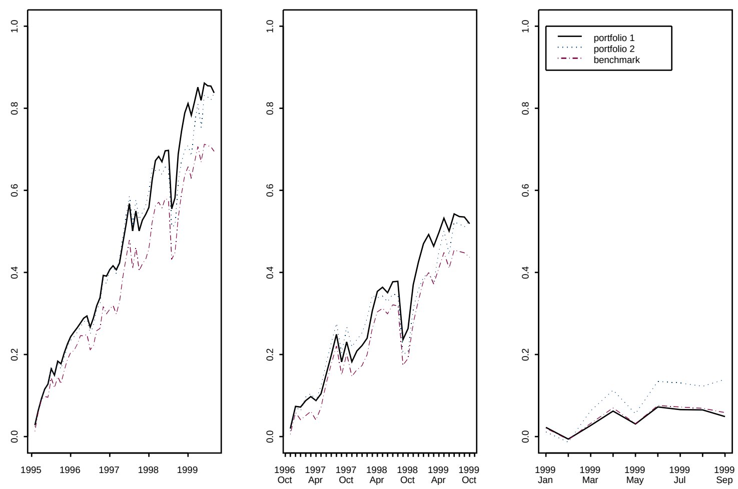

Figure 2: Cumulative returns for 56 months, 3 years and year to date

	Portfolio 1	**Portfolio 2**	**benchmark**
mean returns p.a	17.95%	17.91%	14.95%
st.dev. p.a.	12.45%	13.67%	12.91%
cum. returns			
56 months (02.95-09.99)	83.79%	83.74%	69.51%
3 years (10.96-09.99)	51.89%	52.74%	43.69%
year to date	4.88%	13.96%	5.82%
Sharpe ratio	0.346	0.314	0.267
success rate	64.3%	62.5%	

Table 3: Performance of portfolio 1 and 2 and the benchmark (MSCI World index)from February 1995 until September 1999

5 Conclusion

This paper has shown how multivariate time series forecasts can be used for active portfolio management. Portfolio 1 (the global minimum variance portfolio) is based only on the forecasted variance matrix. Portfolio 2 uses variance and mean forecasts; the mean returns are forecasted with exogenous variables. Evaluation of the portfolio returns with the MSCI World index by different criteria shows that both portfolios perform better than the benchmark for in-the sample period. Transactions costs and a risk-free asset are not included in the current analysis.

Portfolio 1 has negative active weights for the Pacific index in the first nine months of 1999 because of a larger volatility in this region. This leads to a poorer performance than the benchmark for this period. This portfolio has a greater *Sharpe* ratio than portfolio two and will be preferred by risk averse investors. It dominates on the benchmark because it has greater mean returns and a smaller standard deviation over the back-testing period.

Portfolio 2, based on more information, has greater cumulative returns from October 1996 until September 1999 (three years comparison) and performs much better than portfolio 1 in the last nine months (year to date comparison). Both strategies lead to success rates of more than 62%, which means that active portfolio management is not just good or bad luck and that time series forecasts can be successfully applied for market timing.

The disadvantage of the two strategies for portfolio construction is that the weights are very sensitive to changes in the variance-covariance matrix and small differences in the prediction can lead to big differences in the active weights. A Bayesian approach where prior information could be included in the model can be used to reduce the variability of the optimal portfolio weights (see Pojarliev and Polasek, 2000).

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