The VAR–VARCH model: A Bayesian approach

Wolfgang Polasek

and

Hideo Kozumi

Institute of Statistics and Econometrics

University of Basel

Holbeinstrasse 12, CH-4051 Basel, Switzerland

Abstract

In this paper, we develop a combined Bayesian vector autoregressive and conditional heteroskedasticity (VAR–VARCH) models. A Gibbs sampling approach is suggested for the univariate and multivariate VAR–VARCH models. Using a random coefficient formulation it is shown that full conditional distributions are derived in closed analytical forms. The method is applied to monthly exchange rate data, Swiss Francs and Deustch Marks to U.S. Dollars.

Keywords: exchange rates, full conditional distributions, Gibbs sampling, random coefficients, VAR–VARCH models.

1 Introduction

ARCH (autoregressive conditional heteroskedasticity) models have obtained considerably attention in the analysis of financial time series since their introduction by Engle (1982). They are used to capture the tendency for volatility clustering, i.e., for the tendency of large (small) price changes to be followed by other large (small) price changes. There are now more than several hundred papers discussing theoretical properties of ARCH models as well as empirical applications (see Bollerslev *et al.*, 1992 for a good review).

From a Bayesian point of view, Geweke (1988, 1989a, 1989b) analysed univariate ARCH models via Monte Carlo integration. Instead of Monte Carlo integration method, Korn (1993) and Polasek (1993) applied the Gibbs sampler to univariate ARCH models using a random coefficient (RC) formulation of Tsay (1988).

As an extension of univariate results, it is sometimes necessary to analyse volatility clustering within a multivariate framework. Along this line, several authors have proposed vector ARCH (VARCH) models (see, for example, Diebold and Nerlove, 1989 and Bollerslev, 1990). However, there are only few Bayesian analyses in this field.

In this paper, we develop a vector autoregressive–VARCH (VAR–VARCH) model which is suitable for a Markov chain Monte Carlo simulation. Using the random coefficient model as parameterization for the ARCH models, we can derive all full conditional distributions in closed analytical forms, so it is easy to apply the Gibbs sampler. The plan of this paper is as follows. In section 2, we describe a model using a RC formulation. In section 3, full conditional distributions are derived, which are necessary to apply the Gibbs samplers. Section 4 illustrates our model with foreign exchange rate data. Conclusions are given in section 5.

2 The models

2.1 The AR–ARCH (p, q) model

Univariate ARCH models with AR structure in the mean are written as

$$y_t = \beta_0 + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + e_t, \quad e_t|I_t \sim N(0, h_t), \tag{1}$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_q e_{t-q}^2, \tag{2}$$

where y_t is the observation at time t ($t = 1, \ldots, T$) and I_t is the information available at time t. The model in (1) and (2) has an observationally equivalent RC representation given by

$$y_t = x_t'\beta + z_t'\gamma_t + u_t, \quad u_t \sim N(0, \omega), \tag{3}$$

where $x_t = (1, y_{t-1}, \ldots, y_{t-p})'$, $\beta = (\beta_0, \ldots, \beta_p)'$, $z_t = (e_{t-1}, \ldots, e_{t-q})'$, and $\gamma_t \sim N(0, \Sigma)$. Also, u_t and γ_t are assumed to be mutually independent. It is easily verified that the model (3) has the following moments

$$\mathrm{E}(e_t) = 0, \tag{4}$$

$$\mathrm{var}(e_t) = \omega + z_t'\Sigma z_t. \tag{5}$$

If Σ is a full symmetric matrix, the model is called augmented ARCH model (see Bera *et al.*, 1992 and Polasek, 1993). When Σ is a diagonal matrix, the model (3) reduces to the usual ARCH model in (1) and (2), and in such a case $\omega = \alpha_0$ and $\sigma_{ii} = \alpha_i$ ($i = 1, \ldots, p$) where σ_{ii} is the i-th diagonal element of Σ. The model (3) was analysed by Korn (1993) and Polasek (1993).

2.2 The VAR–VARCH (p, q) model

We extend the univariate ARCH model given by (3) to multivariate setting. Let us consider a VAR(p) model of dimension of M (see, e.g., Lütkepohl, 1993) with time dependent (heteroskedastic) covariance matrices:

$$y_t = a_0 + A_1 y_{t-1} + \cdots + A_p y_{t-p} + e_t, \quad e_t|I_t \sim N(0, H_t), \tag{6}$$

where $y_t = (y_{1t}, \ldots, y_{Mt})'$ is an $M \times 1$ vector of observed time series at time t, A_i ($i = 1, \ldots, p$) are fixed $M \times M$ coefficient matrices, $a_0 = (a_{01}, \ldots, a_{0M})'$ is a fixed $M \times 1$ vector of intercept terms, and $e_t = (e_{1t}, \ldots, e_{Mt})'$ is an $M \times 1$ vector of error terms.

The above model is rewritten compactly as

$$y_t = B'x_t + e_t, \tag{7}$$

where $x_t = (1, y'_{t-1}, \ldots, y'_{t-p})'$,

$$B = \begin{pmatrix} a_{10} & \ldots & a_{M0} \\ a_{11} & \ldots & a_{M1} \\ \ldots & \ldots & \ldots \\ a_{1p} & \ldots & a_{Mp} \end{pmatrix},$$

and a_{mi} ($i = 1, \ldots, p$, $m = 1, \ldots, M$) is the m-th $M \times 1$ vector of $A_i = (a_{1i}, \ldots, a_{Mi})'$. In order to extend univariate ARCH models to a multivariate setting, various parameterizations in H_t are proposed (see, for example, Bollerslev *et al.*, 1988 and Bollerslev, 1990). However, the general definition does not exist. Therefore, for simplicity, we assume a diagonal structure of H_t:

$$H_t = \text{diag}(h_{1t}, \ldots, h_{Mt}),$$

and the variance elements depend on the past q residuals of the M time series. This simplification will be modified in the following subsection.

2.3 The VAR–VARCH(p, q) model in RC form

We write the above formulated VAR–VARCH model in a random coefficient form:

$$\begin{aligned} y_t &= B'x_t + e_t, \\ &= B'x_t + \Gamma'_t z_t + u_t, \end{aligned} \tag{8}$$

where $B'x_t$ stands for the VAR component of the VAR–VARCH model and $\Gamma'_t z_t$ for the RC component defined by

$$\begin{aligned} z_t &= (e'_{t-1}, \ldots, e'_{t-q})', \\ \Gamma_t &= (\gamma_{1t}, \ldots, \gamma_{Mt}), \end{aligned}$$

or b that $_{t}$ $_{inn}$ tel teerroriy $_{tcmd.incTTTctcmpnn}$

and q is the order of the VARCH model. The error term is normally distributed with seemingly unrelated (SUR) type covariance matrix:

$$u_t \sim N(0, \Omega), \tag{9}$$

where Ω is an $M \times M$ positive definite matrix. The random coefficient matrix Γ_t has now the following stochastic structures:

$$\text{vec}\Gamma_t \sim N(0, \Sigma),$$

and Σ is a $M^2 q \times M^2 q$ block diagonal matrix:

$$\Sigma \;=\; \begin{pmatrix} \Sigma_1 & & 0 \\ & \ddots & \\ 0 & & \Sigma_M \end{pmatrix}.$$

For a parsimonious model, we further assume that each Σ_m is a block diagonal matrix of dimension $Mq \times Mq$, i.e.,

$$\Sigma_m = \begin{pmatrix} \Sigma_{m1} & & 0 \\ & \ddots & \\ 0 & & \Sigma_{mq} \end{pmatrix},$$

where Σ_{mi} is an $M \times M$ positive definite matrix.

Thus, the RC–component of model (8) has the moments

$$\begin{aligned} \text{E}(\Gamma_t' z_t) &= 0, \\ \text{var}(\Gamma_t' z_t) &= \text{diag9flflTD9CTf9flTD9flTc9-z} \end{aligned}$$

3 Full conditional distributions

We now show the model under study has convenient conditional structure to apply the Gibbs sampler. The Bayesian VAR–VARCH(p, q) model is given as

$$y_t = B'x_t + \Gamma_t' z_t + u_t, \tag{10}$$

and prior distributions are chosen from the families of normal and Wishart distributions, hence

$$\begin{aligned} \text{vec}B &\sim N(\beta_*, H_*), \\ \Omega^{-1} &\sim W_M(\Omega_*, n_*), \\ \text{vec}\Gamma_t &\sim N(0, \Sigma), \\ \Sigma_{mi}^{-1} &\sim W_M(\Sigma_{mi*}, \nu_{mi*}), \end{aligned} \tag{11}$$

where all of the hyper-parameters (which are denoted with a star) are known a priori. Thus, we can derive the following full conditional distributions (f.c.d.'s) for the Gibbs simulation process.

a) The f.c.d. for the VAR coefficients vec$B$:

The model (10) can be rewritten as

$$\begin{aligned} u_t &= y_t - (\mathrm{I}_M \otimes x_t')\text{vec}B - \begin{pmatrix} \gamma_{1t}' \\ \vdots \\ \gamma_{Mt}' \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \vdots \\ y_{t-q} \end{pmatrix} + \begin{pmatrix} \gamma_{1t}' \\ \vdots \\ \gamma_{Mt}' \end{pmatrix} \begin{pmatrix} \mathrm{I}_M \otimes x_{t-1}' \\ \vdots \\ \mathrm{I}_M \otimes x_{t-q}' \end{pmatrix} \text{vec}B, \\ &= y_t - \begin{pmatrix} \gamma_{1t}' \\ \vdots \\ \gamma_{Mt}' \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \vdots \\ y_{t-q} \end{pmatrix} - \left[(\mathrm{I}_M \otimes x_t') - \begin{pmatrix} \gamma_{1t}' \\ \vdots \\ \gamma_{Mt}' \end{pmatrix} \begin{pmatrix} \mathrm{I}_M \otimes x_{t-1}' \\ \vdots \\ \mathrm{I}_M \otimes x_{t-q}' \end{pmatrix} \right] \text{vec}B, \\ &= \tilde{y}_t - \tilde{x}_t' \text{vec}B, \end{aligned}$$

where $\otimes$ denotes the kronecker product. Denoting $\tilde{Y} = (\tilde{y}_1', \ldots, \tilde{y}_T')'$ and $\tilde{X} = (\tilde{x}_1, \ldots, \tilde{x}_T)'$, we can derive the f.c.d.,

$$p(\text{vec}B|\theta^c, data) = N(\beta_{**}, H_{**}), \tag{12}$$

where

$$\begin{aligned} H_{**}^{-1} &= \tilde{X}'(\mathrm{I}_T \otimes \Omega)^{-1}\tilde{X} + H_*^{-1}, \\ \beta_{**} &= H_{**}\{\tilde{X}'(\mathrm{I}_T \otimes \Omega)^{-1}\tilde{Y} + H_*^{-1}\beta_*\}, \end{aligned}$$

and θ^c denotes a vector of all parameters save the arugmen

d) The f.c.d. for the ARCH coefficient matrix Σ_{mi}^{-1}:

$$\begin{aligned} p(\Sigma_{mi}^{-1}|\theta^c, data) &\propto |\mathrm{I}_T \otimes \Sigma_{mi}|^{-\frac{1}{2}} \exp\Big\{-\frac{1}{2}\sum_{t=1}^{T}\gamma'_{mt,i}\Sigma_{mi}^{-1}\gamma_{mt,i}\Big\} \\ &\quad \times|\Sigma_{mi}^{-1}|^{\frac{1}{2}(\nu_{mi*}-M-1)}\mathrm{etr}\Big\{-\frac{1}{2}\Sigma_{mi*}\Sigma_{mi}^{-1}\Big\}, \\ &= W_M(\Sigma_{mi**}, \nu_{mi**}), \end{aligned} \tag{15}$$

where

$$\begin{aligned} \nu_{mi**} &= \nu_{mi*} + T, \\ \Sigma_{mi**} &= \Sigma_{mi*} + \sum_{t=1}^{T}\gamma_{mt,i}\gamma'_{mt,i}, \end{aligned}$$

and $\gamma_{mt} = (\gamma'_{mt,1}, \ldots, \gamma'_{mt,q})'$.

The Gibbs sampling technique is explained in Gelfand and Smith (1990). The iteration process consists of drawing random numbers from the distributions in (14), (13), (15) and (12) in that order.

4 Illustrative examples

We present two examples to illustrate our models. The first example analyses a data set of monthly exchange rate for Swiss Franc/Dollar (SFr/USD) and Deutsche Mark/Dollar (DM/USD) for the period October 1973 to March 1988 using univariate ARCH models (i.e., $M = 1$). (The SFr/USD and DM/USD are shown in Figure 1.) The second example estimates VAR-VARCH models for the same two exchange rates. All time series are in log difference form, giving 186 observations of monthly growth rates.

The results given below are based on the following Gibbs sampling scheme. We used multiple parallel chains with an overdispersed initial sample to be able to apply the convergence diagnostic proposed by Gelman and Rubin (1992). This diagnostic $\sqrt{\hat{R}}$ estimates the potential scale reduction, and $\hat{R}$ is the ratio of the current variance estimate to the within-sequence variance with a factor to account for the extra variance of the Student's t distribution. To initialize the simulation, we first ran a single chain with 1,000 iterations, computed for each parameters (θ_i) approximate posterior means

$\bar{\theta}_i$ and standard deviations σ_i based on the last 500 iterations, and then generated initial values from $t(\bar{\theta}_i, \sigma_i^2)$ with degrees of freedom 4. After obtaining initial values, we ran 10 parallel chains with 6,000 iterations, and the last 1,000 samples for each chain are used for parameter estimation.

The specification of the hyper-parameters in (11) are rather noninformative. The prior parameters of the VAR component are

$$\beta_* = 0, \qquad H_*^{-1} = \mathrm{diag}(0.01, \ldots, 0.01),$$

which can be viewed as a tightness prior around zero. The prior for the SUR–type covariance matrix in the VAR component is

$$n_* = M + 2, \qquad \Omega_* = \mathrm{diag}(0.01, \ldots, 0.01).$$

The prior parameters for ARCH coefficients matrix Σ_{mi} are

$$\nu_{mi*} = M + 2, \qquad \Sigma_{mi*} = \mathrm{diag}\left(\frac{0.8(q+1-i)}{0.5q(q+1)}, \ldots, \frac{0.8(q+1-i)}{0.5q(q+1)}\right).$$

This specification follows a linear decay pattern with the properties $\mathrm{E}(\Sigma_{mi}) = \Sigma_{mi*}$ and $\sum_{i=1}^{q} \frac{0.8(q+1-i)}{0.5q(q+1)} = 0.8$ for each of the m components.

4.1 Example 1: univariate analysis

In the first example, we estimated the model (3) without the intercept for both exchange rates, SFr/USD and DM/USD, and for various combination of orders, p and q. For the model selection, we have calculated the conditional predictive ordinate (CPO) based on the forecasting predictive density for all time points $t = 1, \ldots, T$. The CPO is explained in the appendix. Since the CPO's for SFr/USD are very similar to those for DM/USD, the CPO–plots only for SFr/USD are shown in Figure 2. From Figure 2, it is seen that the medians of the CPO–plots are very similar, and that the CPO's are fairly left skewed. Also, we see that the ARCH models with high order tend to produce large values of CPO's. These large values occur for the period 1980–1982 and 1985. As seen from Figure 1, exchange rates are rather unstable in these periods. It may be considered that ARCH models with high order are suitable for these periods. However, since ARCH parameters for high order are not significant, we choosed an AR–ARCH(1,1) model as the sufficient model.

In Table 1 the estimated potential scale reductions of the Gelman–Rubin diagnostics are reported. All values are close to 1.0, indicating that simulated values are close to the target distribution.

Posterior means and standard deviations are shown in Table 2. It can be seen that ARCH parameters are significant both in SFr/USD and DM/USD, but their magnitudes are small as it was also reported in Domowitz and Hakkio (1985). ("Significant" in our interpretation means that the posterior mean is more than two posterior standard deviations away from zero.) Using monthly exchange rate data from March 1980 to January 1985, Baillie and Bollerslev (1989) reported that there are no significant ARCH effects. Compared with our results, we conclude that their finding is possibly due to a small sample size. Moreover, the estimates for SFr/USD and DM/USD are very similar. This result is consistent with a visual inspection of Figure 1. Figure 5 shows that the conditional variances of the DM/USD are smaller than the ones of the SFr/USD.

In Figure 3, the kernel density estimates for the marginal posterior distributions are shown. The distributions for β_1 and ω appear to be fairly symmetric. Not surprisingly, the distributions for σ_{11} are skewed to the right (reflecting the contribution to the conditional variance of the time series).

4.2 Example 2: multivariate analysis

Based on the previous results, we estimated the VAR–VARCH(1,1) model using the SFr/USD and DM/USD exchange rates. The CPO–plots of the VAR–VARCH model up to order $(p, q) = (2, 2)$ are shown in Figure 4. As in the previous example, it can be seen that the predictive performance of the VAR–VARCH(1,1) model is about the same as for the VAR–VARCH(2,2) model. There is no unique answer for the model choice from the CPO–plots.

Estimated potential scale reductions for the Gelman–Rubin diagnostics are given in Table 3, indicating that all values are close to one. Parameter estimates for the VAR–VARCH(1,1) model are as follows:

VAR component:

$$\begin{array}{rcccl} \mathrm{SFr/USD}_t & = & 0.4338\,\mathrm{SFr/USD}_{t-1} & - & 0.1595\,\mathrm{DM/USD}_{t-1}, \\ & & (0.1532) & & (0.1723) \\ \mathrm{DM/USD}_t & = & 0.1025\,\mathrm{SFr/USD}_{t-1} & + & 0.2082\,\mathrm{DM/USD}_{t-1}, \\ & & (0.1506) & & (0.1641) \end{array}$$

$$\Omega \times 10^3 = \begin{pmatrix} 0.8757 & \\ (0.0987) & \\ 0.6798 & 0.7431 \\ (0.0816) & (0.0819) \end{pmatrix},$$

VARCH component:

$$\Sigma_1 = \begin{pmatrix} 0.1345 & \\ (0.0663) & \\ -0.0857 & 0.1373 \\ (0.0637) & (0.0704) \end{pmatrix}, \qquad \Sigma_2 = \begin{pmatrix} 0.1052 & \\ (0.0462) & \\ -0.0645 & 0.1102 \\ (0.0451) & (0.0517) \end{pmatrix},$$

where posterior standard deviations are given in parentheses.

The estimates of the VAR component of the model show that only the AR coefficient for $\mathrm{SFr/USD}_{t-1}$ is significant. Moreover, all elements of Ω are highly significant, and a high correlation is found. This finding agrees with a visual inspection from Figure 1.

The results of the VARCH component show that both diagonal elements of Σ_2 are significant and their magnitudes are almost the same. This finding indicates that the SFr/USD helps to explain the volatiliy movement of the DM/USD. However, the second diagonal element of Σ_1 is not significant, and this means that the DM/USD does not explain the volatility movement of the SFr/USD exchange rate on a monthly basis. Off-diagonal elements of Σ_1 and Σ_2 are negative, but not significant.

In Figure 5, the conditional variances both for univariate and multivariate models are plotted. It can be seen, for the SFr/USD, that the conditional variance for univariate model is slightly larger than that for multivariate model. In contrast to the SFr/USD exchange rate, the result of the DM/USD shows that the conditional variance for the univariate model is smaller than that for the multivariate model over the whole time period.

Figure 6 displays the kernel density estimates for the marginal posterior distributions. It can be seen that ARCH coefficients are skewed to the right for diagonal elements, and to the left for off-diagonal elements with all modal values distinctively different from zero.

5 Conclusions

In this paper, we have analysed Bayesian VAR–VARCH models for exchange rates using a random coefficient formulation. We have shown that all full conditional distributions can be derived in a closed form. This allows an easy implementation of the Gibbs sampling routine since random variables can be easily drawn from the full conditional distributions.

Using monthly exchange rate data, we have composed models with different orders our both for univariate and multivariate models. The univariate analysis showns that the ARCH effects can be clearly seen both in the SFr/USD and the DM/USD series, but that the effects are rather small. Also, the posterior distributions for ARCH parameters are skewed to the right. The multivariate analysis shows that the SFr/USD helps to explain the volatility movement of the DM/USD, but the reverse is not true. We conclude that a VAR–VARCH model reduces and re-allocates the effects of conditional variances. The conditional variances of univariate models are smaller than those of the multivariate models in the DM/USD.

Appendix

The conditional predictive ordinate (CPO) based on the forecasting predictive distribution of y_t given $(y_1, \ldots, y_{t-1})$ is denfined as

$$\begin{aligned} f(y_t|y_1,\ldots,y_{t-1}) &= \frac{f(y_1,\ldots,y_t)}{f(y_1,\ldots,y_{t-1})}, \\ &= \frac{\int \frac{f(y_1,\ldots,y_t|\theta)}{f(y_1,\ldots,y_T|\theta)}\pi(\theta|y_1,\ldots,y_T)d\theta}{\int \frac{f(y_1,\ldots,y_{t-1}|\theta)}{f(y_1,\ldots,y_T|\theta)}\pi(\theta|y_1,\ldots,y_T)d\theta}, \end{aligned} \tag{16}$$

where θ is a parameter vector, $f(\cdot|\theta)$ is the likelihood function, and $\pi(\theta|\cdot)$ is the posterior density function. The CPO given by (16) suggests what values of y_t are likely, given that the model was fitted to the observations $y_1, \ldots, y_{t-1}$, and it is possible to see whether the observation supports the model. Using the output from the Gibbs sampler, $\theta^{(i)}$, $i = 1, \ldots, N$, an approximation to (16) may be obtained by Monte Carlo integration

$$\hat{f}(y_t|y_1,\ldots,y_{t-1}) = \frac{\frac{1}{N}\sum_{i=1}^{N} \frac{f(y_1,\ldots,y_t|\theta^{(i)})}{f(y_1,\ldots,y_T|\theta^{(i)})}}{\frac{1}{N}\sum_{i=1}^{N} \frac{f(y_1,\ldots,y_{t-1}|\theta^{(i)})}{f(y_1,\ldots,y_T|\theta^{(i)})}}.$$

References

[1] Baillie, R.T. and Bollerslev, T. (1989). The message in daily exchange rates: A conditional–variance tale, *Journal of Business and Economic Statistics 7*, 297–305.

[2] Bera, A.K., Higgins, M.L., and Lee, S. (1992). Interaction between autocorrelation and conditional heteroscedasticity: a random–coefficient approach, *Journal of Business and Economic Statistics 10*, 133–142.

[3] Bollerslev, T. (1990). Modelling the coherence in short–run nominal exchange rates: A multivariate generalized ARCH approach, *Review of Economics and Statistics 72*, 498–505.

[4] Bollerslev, T., Engle, R.F., and Wooldridge, J.M. (1988). A capital asset pricing model with time varying covariances, *Journal of Political Economy 96*, 116–131.

[5] Bollerslev, T., Chou, R.Y., and Kroner, K.F. (1992). ARCH modeling in finance, *Journal of Econometrics 52*, 5–59.

[6] Diebold, F.X. and Nerlove, M. (1989). The dynamics of exchange rate volatility: A multivariate latent factor ARCH model, *Journal of Applied Econometrics 4*, 1–21.

[7] Domowitz, I. and Hakkio, C.S. (1985). Conditional variance and the risk premium in the foreign exchange market, *Journal of International Economics 19*, 47–66.

[8] Engle, R.F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation, *Econometrica 50*, 987–1008.

[9] Gelfand, A.E. and Smith, A.F.M. (1990). Sampling based approaches to calculating marginal densities, *Journal of the American Statistical Association 85*, 398–409.

[10] Gelman, A. and Rubin, D.B. (1992). Inference from iterative simulation using multiple sequences, *Statistical Science 7*, 63–86.

[11] Geweke, J. (1988). Exact inference in models with autoregressive conditional heteroskedasticity, in :E. Berndt, H. White, and W. Barnett, eds., *Dynamic econometric modeling*, (Cambridge University Press, Cambridge) 73–104.

[12] Geweke, J. (1989a). Exact predictive densities in linear models with ARCH disturbances, *Journal of Econometrics 44*, 307–325.

[13] Geweke, J. (1989b). Bayesian inference in econometric models using Monte Carlo integration, *Econometrica 57*, 1317–1339.

[14] Korn, O. (1993). A Gibbs sampler for Bayesian ARCH models, WWZ discussion paper, University of Basel.

[15] Lütkepohl, H. (1993). Introduction to multiple time series analysis 2nd ed., Springer–Verlag, Berlin.

[16] Polasek, W. (1993). Bayesian augmented ARCH models, WWZ discussion paper, University of Basel.

[17] Tsay, R.S. (1987). Conditional heteroscedastic time series models, *Journal of the American Statistical Association 82*, 590–604.

Table 1: Rubin–Gelman diagnostics $\sqrt{\hat{R}}$ for example 1.

	β_1	ω	σ_{11}
SFr/USD	1.0006	1.0003	1.0169
DM/USD	1.0004	1.0002	1.0145

Table 2: Parameter estimates for example 1.

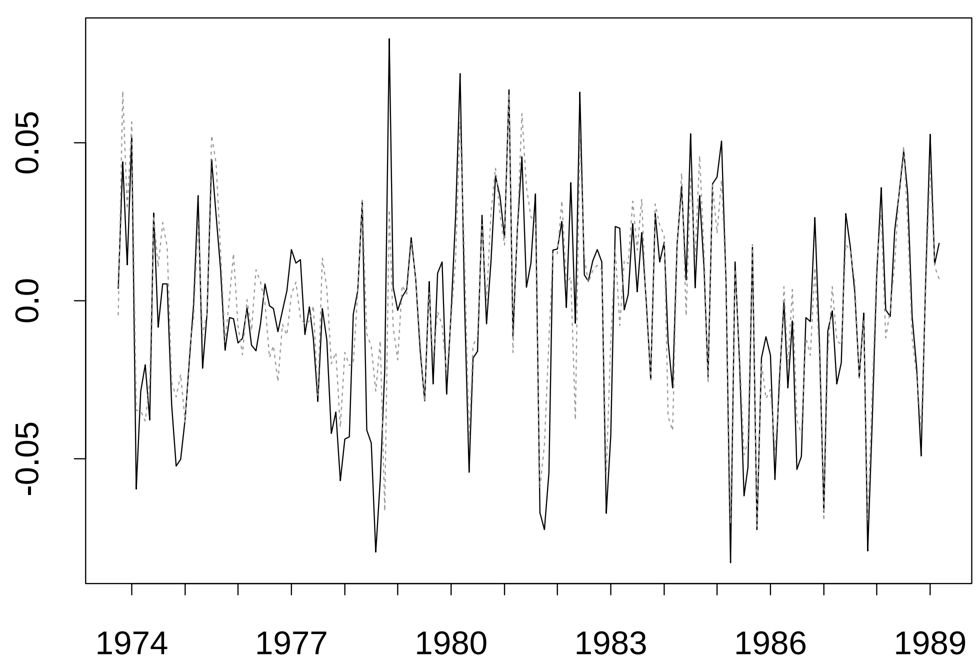
	SFr/USD		DM/USD	
	mean	sd.v	mean	sd.v
β_1	0.3522	0.0792	0.3291	0.0796
$\omega \times 10^3$	0.8833	0.1014	0.7206	0.0809
σ_{11}	0.1365	0.0599	0.1223	0.0512

Table 3: Rubin–Gelman diagnostics $\sqrt{\hat{R}}$ for example 2.

$B(1,1)$	$B(1,2)$	$B(2,1)$	$B(2,2)$	$\Omega(1,1)$	$\Omega(1,2)$	$\Omega(2,2)$
1.0006	1.0005	1.0007	1.0005	1.0003	1.0001	1.0001

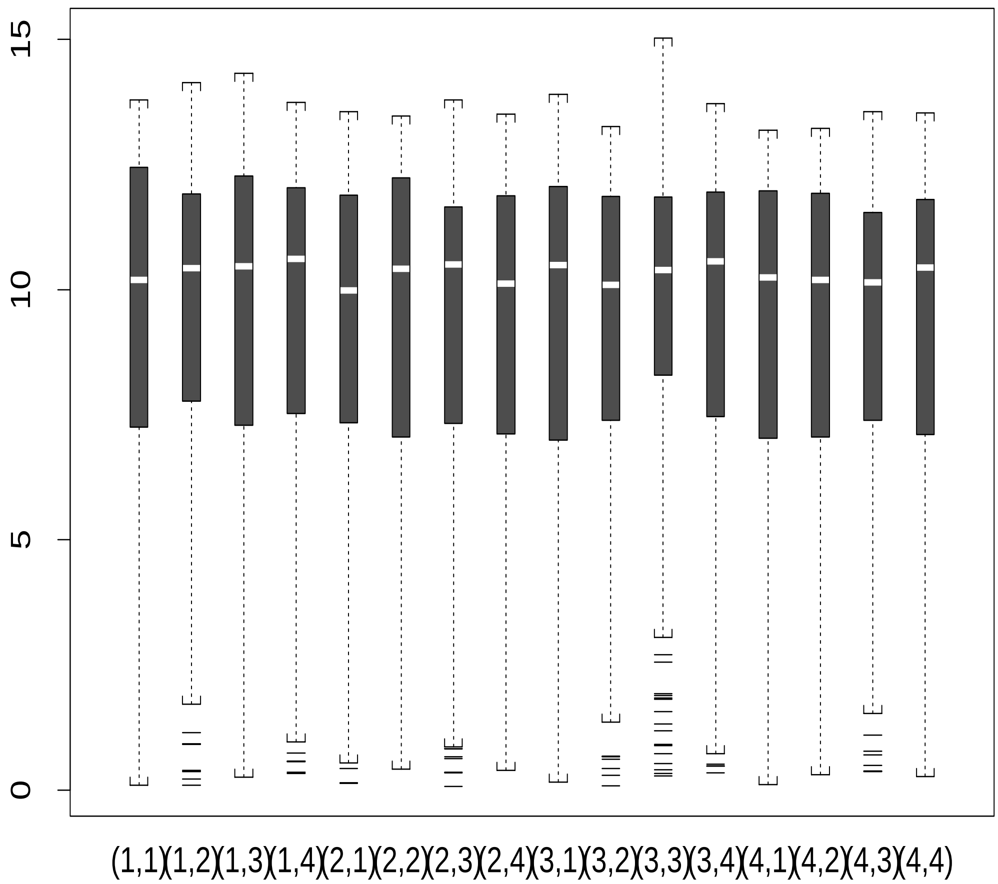
$\Sigma_1(1,1)$	$\Sigma_1(1,2)$	$\Sigma_1(2,2)$	$\Sigma_2(1,1)$	$\Sigma_2(1,2)$	$\Sigma_2(2,2)$
1.0276	1.0339	1.0293	1.0195	1.0219	1.0153

Figure 1: SFr/USD and DM/USD rates.



SFr/USD (solid line) and DM/USD (dotted line).

Figure 2: CPO plots for AR–ARCH models.



(p, q)

Figure 3: Marginal posterior densities for the AR–ARCH(1,1) model.

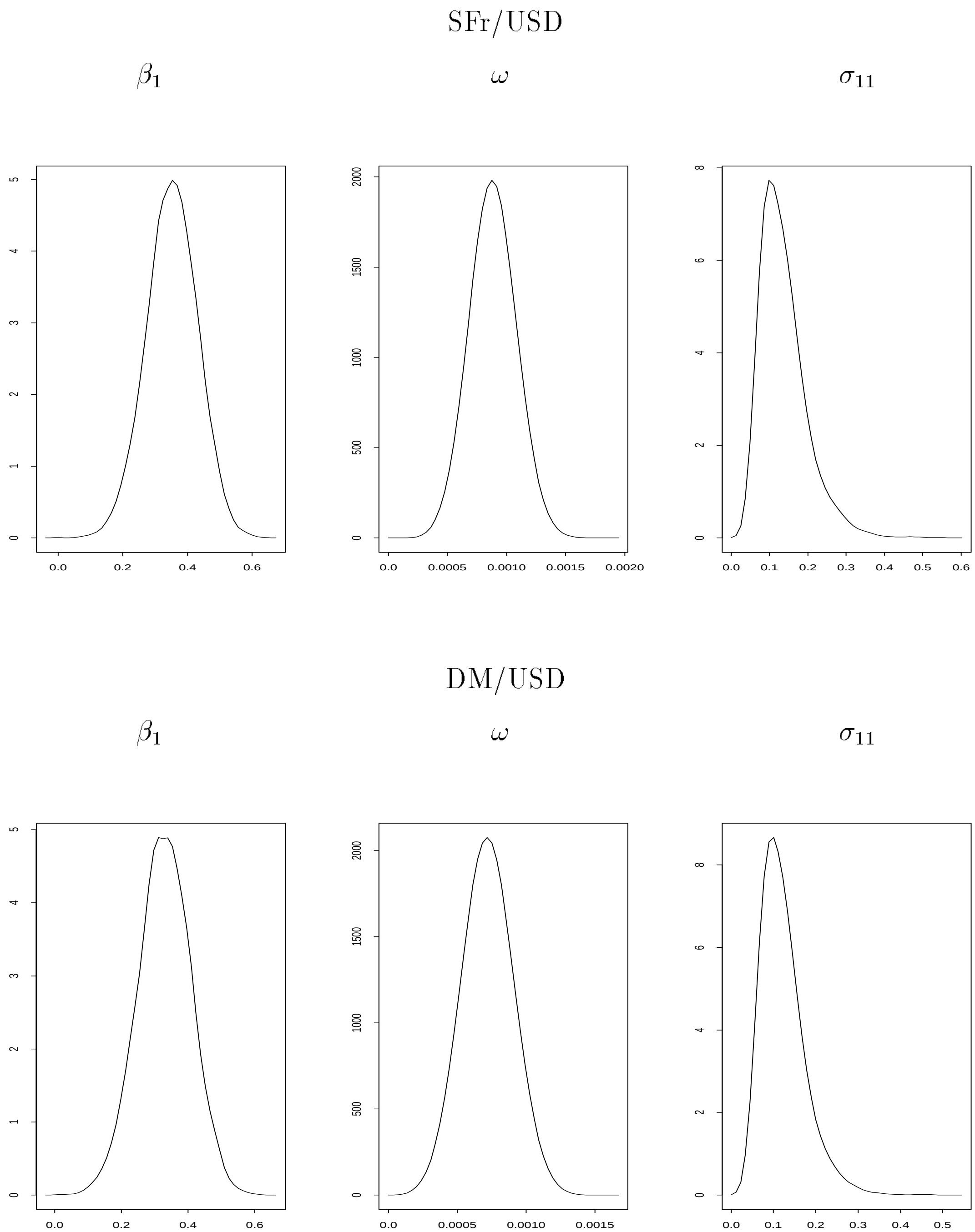


Figure 4: CPO plots for VAR–VARCH models.

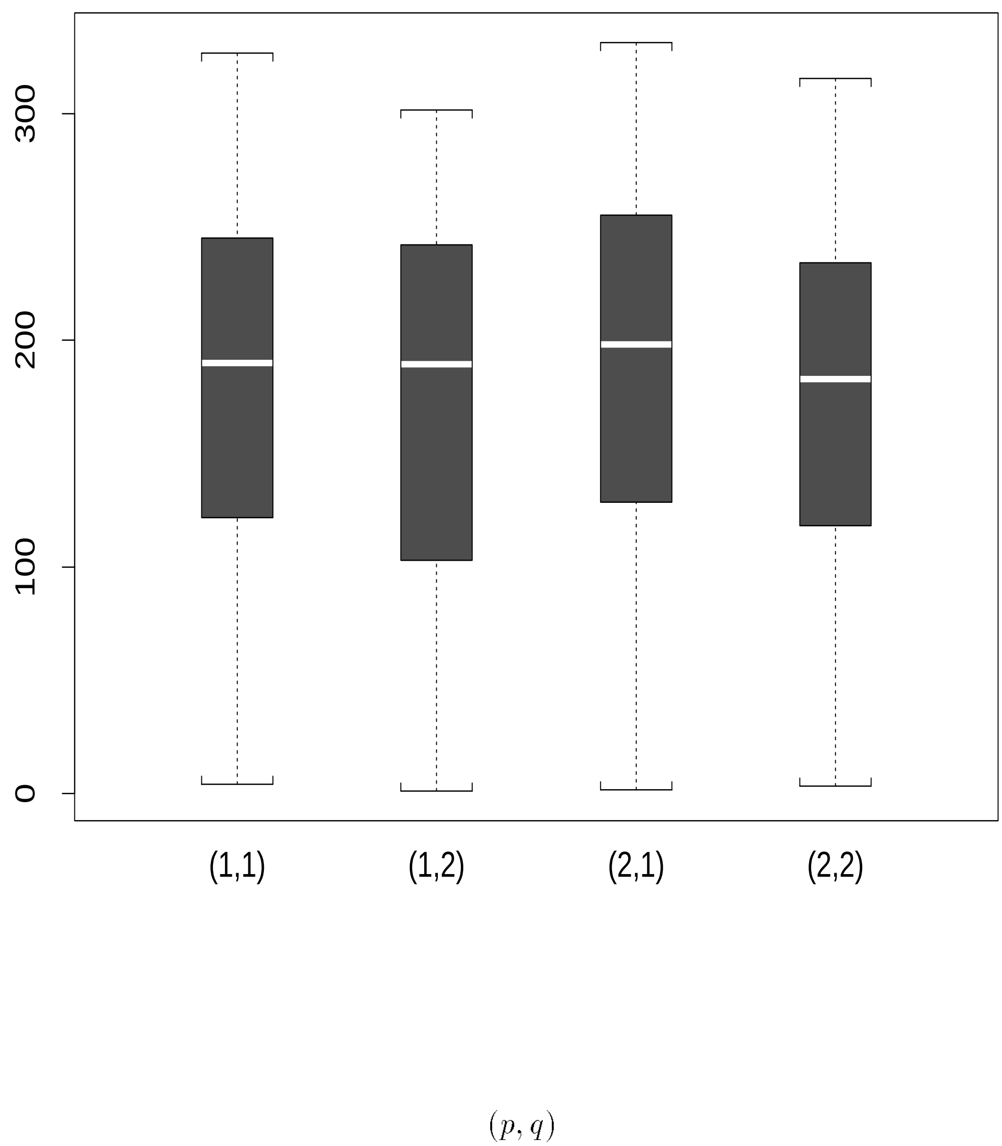
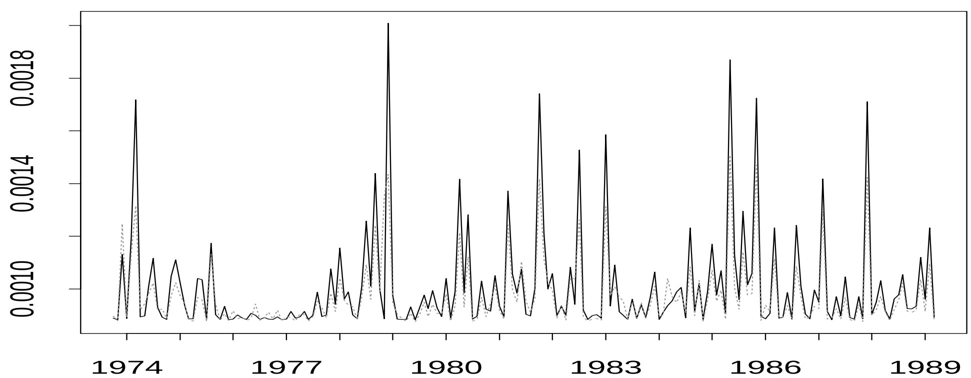
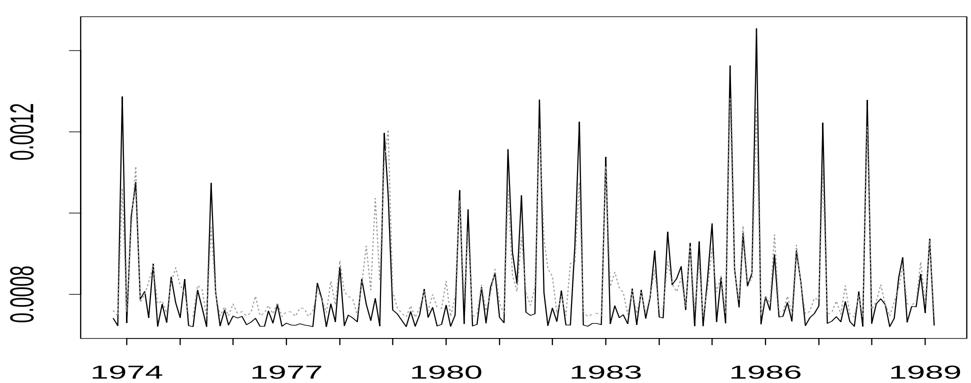


Figure 5: Conditional variances for univariate and multivariate models.

SFr/USD



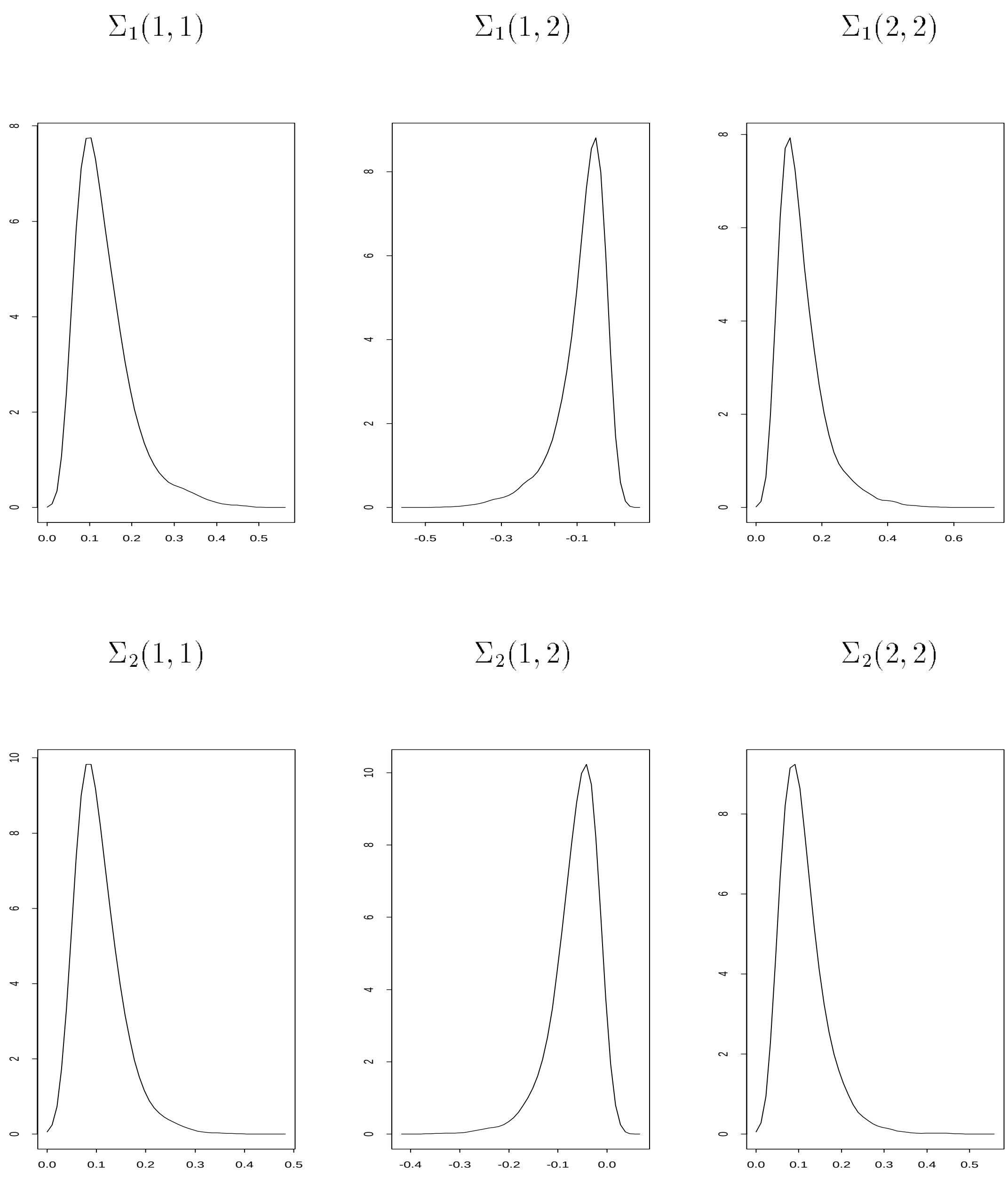
DM/USD



Univariate model (solid line) and multivariate model (dotted line).

Figure 6: Marginal posterior densities for the VAR–VARCH(1,1) model.

$B(1,1)$ $B(2,1)$ $B(1,2)$ $B(2,2)$


$\Sigma_1(1,1)$
$\Sigma_1(1,2)$
$\Sigma_1(2,2)$
$\Sigma_2(1,1)$
$\Sigma_2(1,2)$
$\Sigma_2(2,2)$
0.0 0.1 0.2 0.3 0.4 0.5
-0.5 -0.3 -0.1
0.0 0.2 0.4 0.6
0.0 0.1 0.2 0.3 0.4 0.5
-0.4 -0.3 -0.2 -0.1 0.0
0.0 0.1 0.2 0.3 0.4 0.5