Global European portfolio construction: Does a changing volatility structure matter?

Wolfgang Polasek and Momtchil Pojarliev

Institute of Statistics and Econometrics
University of Basel
Bernoullistrasse 28
CH-4056 Basel
email: Wolfgang.Polasek@unibas.ch

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Abstract

We propose a multivariate time series model to forecast the returns and volatilities of 16 European financial markets. Using the approach of mean-variance portfolios we develop several strategies which are based on the predictions of high-dimensional VAR-GARCH models for future volatilities. We explore the value of volatility timing strategies by making simplification to the forecasting model. One approach for information blocking is based on factor analysis for the returns. Finally we discuss if multivariate volatility timing strategies are successful for beating the benchmark index (the MSCI Europe index).

Key words: multivariate time series models, volatility of financial markets, mean-variance portfolios, factor analysis.

1 Introduction

The European economies have changed dramatically in the 90ies after the break-up of the Soviet Union and the creation of the European monetary union which led to a convergence of macro-economic variables for EU country members. Also, the international financial markets have seen a strong rise due to technological progress and the globalization of markets. Furthermore, an improved information basis due to the internet has changed the amount and speed of information which are relevant for stock and currency exchange markets. New techniques of information porcessing and electronic trading tools have changed the efficiency and transmission mechanism of financial markets.

After the fall of the iron curtain and the creation of the common currency Euro, European countries have become increasingly important for international financial investors. Using the quantitative approach of modern portfolio theory we will explore strategies as how to diversify and control the risk of investments in 15 European countries. It is well known that simple active portfolio management is no guarantee to outperform capitalization weighted benchmarks (see e.g. the recent review of Goldman and Sachs [?]). Therefore new approaches have been developed to overcome the challenge of active portfolio management. Pojarliev and Polasek [?] (PP henceforth) have proposed a new approach using dynamic time series models for constructing a global regional model (Europe, Pacific and North America) with notable success (4% p.a. over the benchmark). Now we face a higher dimensional model where the curse of dimensionality will challenge the econometric estimation process with too many parameters. Therefore intelligent modeling and estimation procedures have to be found to minimize the computational burden and they have to be checked for useful results. We investigate the question to what extend the creation of the common market in Europe and the convergence to a monetary union have affected the volatility structure between financial markets in Europe.

We compare the performance of a special European "factor" portfolio which

assumes a block structure for the variance matrix of the 15 MSCI indices. We address also the question how much information is lost by assuming a constant volatility structure and how does it affect the portfolio performance. We compare a "buy-and-hold" portfolio which is based on the historical variance matrix of the monthly returns with various multivariate time series models. These multivariate VAR-GARCH (vector autoregressive generalized conditional heteroskedasticity) models are used to predict one-step-ahead returns and volatilities which are transformed into optimal portfolio weights. The intention of the paper is to explore the structure of multivariate volatility models on the portfolio performance. If a significant better portfolio return can be obtained by conditional volatility models then a changing volatility structure is important for building high-dimensional portfolios. The performance of the portfolios is compared for the last 1, 2 and 2 1/2 years (until June 2001).
This paper is organized as follows: The next section 2 describes the European stock indices for the portfolio analysis. Section 3 presents the time series models for volatility forecasting and Section 4 explains various strategies for portfolio construction. Section 5 summarizes the results of the portfolio performances and compares them with the benchmark, the MSCI Europe index. The last section concludes. In the appendix we explain the factor analysis approach and we list the descriptive results of the index return for each country.

2 Portfolios for European stock indices

For the European stock portfolio we investigate the monthly returns of the national MSCI indices for Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and Great Britain. As an additional 16th country we have considered Luxembourg and in a next step Greece will be included. The whole time period stretches from January 1990 to July 2001 and is divided into two parts:

an in-sample-period of 9 years (108 observations) until December 1998 which is used as the "training sample" for estimating the models. An out-of-sample period of 12-31 months is used for the portfolio evaluation. Tables 4, 5, and 6 show the descriptive statistics of the monthly MSCI returns for the European markets up to January 2001. From the appendix we see that all index returns are positively skewed and that the kurtosis is largest for the Norway (2.96) and smallest for Finland (0.18). The risk measured by the standard deviation is the largest for the MSCI Finland index (0.0893) and smallest for the MSCI Netherlands index (0.0438). Note that all 25% quartiles are negative and Austria is the only country where the mean return is negative (but the median is slightly positive). All monthly return distributions are positively skewed and only UK has a negative excess kurtosis.

2.1 The multivariate approach

Our approach will examine the value of volatility timing for quantitative portfolios. We determine the weights of the optimal global mean-variance (GMV) portfolios by the (predicted) variance-covariance matrix. Therefore the forecasting performance of the multivariate volatility model will be reflected in the optimized weights and determines directly the portfolio performance. PP [?] have iteratively forecasted the optimal weights for the MSCI North America, MSCI Europe and MSCI Pacific indices from the predicted covariance matrix of a multivariate GARCH model (specifically the BEKK model of Engle and Kroner [?]) and the so obtained GMV portfolio beats the MSCI World index (see PP [?]).
The multivariate time series approach for portfolio construction is more difficult to implement for higher dimensional portfolios: An increasing large number of parameters can lead to less reliable estimates especially if the time series are short. A simple way to solve the dimensionality problem is to divide the covariance matrix of the 15 MSCI indices into three uncorrelated blocks and to assume a zero covariance structure between those blocks. The country blocks are selected by applying pure statistical methods or by using

economic reasoning. In the appendix we show how to use factor analysis for the classification of European countries into 3 blocks. Such an analysis is also an empirical test if the covariance structure is important for a European portfolio performance: If the portfolio of the returns do not change for the models, then a blocking structure is justified and the volatility flow of information is less global.

2.2 Economic blocks

While factor analysis can be considered as a statistical tool to build country blocks, we can also use economic reasoning and geographical information to construct these blocks. The South European countries (Italy, Spain, Portugal and Ireland) together with Ireland make up the first block. The Central European countries, (Germany, Switzerland, Austria, and France) together with Great Britain form the second block. The third block contains the North European countries (Denmark, Finland, Norway and Sweden) and the Benelux countries, Belgium, Luxembourg and Netherlands. (Note that MSCI Europe index doesn't contain the MSCI Luxembourg index but will in future incorporate Greece.) This portfolio is called global European (GE) portfolio.

3 Volatility forecasts

Previous research (see PP [?] and Polasek and Ren [?]) have shown that multivariate GARCH models can outperform univariate GARCH models considerably if the volatility of stock market indices has to be forecasted. Therefore we use a VAR-GARCH model to predict iteratively the variance matrices as a whole or by country blocks. Using the "training sample" of the last 9 years of our data set, we estimate a VAR(0)-GARCH(1,1) models for each block and obtain the forecasted variance matrix for January 1999. This procedure is repeated 12-31 times, using the moving estimation period of 9 years (108 observations). The estimation results of the parameters are less interesting but can be easily reproduced by program packages like EViews or Splus. Model selection for the multivariate VAR-GARCH portfolio was made by calculating the likelihood values for a whole set of models as can be seen from the next table.

Time Series Model	likelihood value
AR(2) and BEKK(1, 1)	3827
AR(2)-M and BEKK(1, 1), t-distr.	3866
AR(2) and ccc.AGARCH(1, 1)	3958*)
AR(2)-M and ccc.AGARCH(1, 1), t-distr.	3956
AR(2) and ccc.AGARCH(2, 2)	3957
AR(2)-M and ccc.AGARCH(2, 2), t-distr.	3953
AR(2) and ccc.GARCH(1, 1)	3958
AR(2)-M and ccc.GARCH(1, 1), t-distr.	3956
AR(3) and ccc.AGARCH(1, 1)	3957
AR(3)-M and ccc.AGARCH(1, 1), t-distr.	3954
ARMA(2,2) and ccc.AGARCH(1, 1)	3958
ARMA(2,2)-M and ccc.AGARCH(1, 1), t-distr.	3956
AR(2) and prcomp(1, 1)	3789
AR(2)-M and prcomp(1,1),t-distr.	3953

Table 1: Likelihood values for VAR-GARCH-M models for 16 MSCI Europe returns from January 1990 until January 2001. The star (*) denotes the smallest values.

We see from the table 1 that the family of constant correlation coefficient GARCH models (ccc.GARCH) and the asymmetric (threshold) GARCH model (AGARCH) is better fitting than the BEKK model. Also the ARMA terms and principal components are not improving the fit. The asymmetric AR(2)-ccc.AGARCH(1,1) model gives the best fit and will be used for the multivariate portfolio construction.

The GARCH-M model is defined simply by taking the conditional variance σ_t^2 as an contemporaneous regressor in the mean equation.

$$r_t = \mu + \gamma\sigma_t^2 + \epsilon_t, \quad t = 1, ..., T. \tag{1}$$

3.1 Multivariate GARCH models

The S+GARCH model allows the estimation of a variety of multivariate GARCH models. Because of the high dimensionality we have tried to fit simple volatility models, like univariate GARCH models and the most complex model is the BEKK model (which is explained in the appendix). Other models with a small number of parameters are the constant correlation coefficient GARCH model and the principal component model. The constant correlation coefficient GARCH model is defined by decomposing a the conditional variance matrices V_t into standard deviations and correlations, where the correlation matrix is assumed to be constant over time:

$$V_t = D_t R D_t$$

where D_t is a diagonal matrix consisting of the standard deviations $D_t = diag(\sigma_{t1}, ..., \sigma_{tK})$.
The principal component model is defined in similar way by an orthogonal decomposition of the conditional variance matrices V_t but now the orthogonal weights are assumed to be constant over time:

$$V_t = Q D_t^2 Q', \quad t = 1, ..., T, \tag{2}$$

Q is the matrix with the orthogonal weights and is fixed through the variance matrix over the whole time period.

3.2 The asymmetric GARCH model

To predict volatilities we assume asymmetric GARCH models for the index returns and normally distributed errors

$$r_t | I_{t-1} \sim N[\mu, \sigma_t^2] \tag{3}$$

or

$$r_t = \mu + \epsilon_t, \quad t = 1, ..., T, \tag{4}$$

where I_{t-1} is the information set until time $t-1$. We assume a constant mean μ for the returns and for the errors ϵ_t a Gaussian distribution with mean zero and variance σ_t^2. We parameterize the conditional variances by an asymmetric GARCH model of orders p and q, which is denoted as AGARCH(p,q) model and has the form (see e.g. Glosten et al. [?])

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i S_{t-i})\epsilon_{t-i}^2 + \sum_{i=1}^{q}\beta_i \sigma_{t-i}^2 \tag{5}$$

where S_t is the dummy variable for the negative residuals and is defined as

$$S_t = \left\{ \begin{array}{lll} 1 & \text{if} & \epsilon_t < 0; \\ 0 & \text{if} & \epsilon_t \geq 0. \end{array} \right\} \tag{6}$$

The idea behind the AGARCH model (also called in the Splus-GARCH module as a 'threshold' ARCH model) is that asymmetric behaviour of the negative shocks are sources for additional risk.

3.3 The GARCH model with t-distribution

Previous research has shown that the conditional distribution for the error term in the mean equation often has heavier tails than the Gaussian distribution. Therefore we estimate a t-GARCH(1,1) model where the t distribution is used as conditional distribution for the residuals in the mean equation of the t-GARCH model. The degrees of freedom are optimized numerically by the grid method.

4 European portfolio construction

In this section we describe briefly the different portfolio strategies we have used for the European countries, except the factor analysis approach which can be found the appendix.

4.1 The factor portfolio FP and the global European portfolio GE

The factor portfolio and the global European (GE) portfolio are based on the variance matrix forecasts described above, where the covariance between the different blocks are assumed to be zero. The weights are determined by the global mean variance formula from the forecasted covariance matrix $\hat{H}^{-1}$ by

$$w_t = \frac{\hat{H}_{t+1}^{-1}\iota}{\iota'\hat{H}_{t+1}^{-1}\iota}. \tag{7}$$

where ι is vector of ones and $\tilde{H}$ is the predicted variance matrix of the 15 MSCI indices, based on 3 blocks: $\tilde{H} = blockdiag(H_1, H_2, H_3)$. This procedure is repeated 31 times using a rolling sample of 120 observations and estimating the VAR-AGARCH(1,1) for each time point. This is a computationally costly procedure. We have used the S-GARCH module of SPlus [**?**]. Figure 1 plots the estimated weights for the Global European portfolio from January 1999 until January 2001. Note that some of the weights can become negative. In this case we will allow for short sales in the portfolio.

4.2 The buy-and-hold portfolio

If we assume a constant volatility structure then a good portfolio strategy is to use the "historical variance" (HV) matrix to compute the optimal portfolio weights. The weights of the "buy-and-hold" (B&H) portfolio are computed as in equation (7), but instead of the forecasted variance matrix $\hat{H}_{t+1}$ we use the sample variance matrix of the 15 MSCI indices from January 1990 until December 1998. This means that the average historical matrix up to December 1998 is used to determine the weights for January 1999 and then this procedure is repeated for the next 30 months.

The weights are given by Ireland: 7.58%, Italy: 56.37%, Portugal: 5.98%, Spain: -3.74%, Austria: -13.39%, France: -22.13%, Germany: -13.82%, Switzerland: 13.36%, UK (Great Britain): 34.15%, Belgium: 0.94%, Denmark: -1.65%, Finland: -15.07%, Netherlands: 2.85%, Norway: 6.18% and Sweden: 42.39%. Note the extremely large weights for Italy (56%), Great Britain (34%) and Sweden (42%).

4.3 The GARCH and the block-GARCH portfolio

The returns of all 15 country (MSCI index) returns are estimated with a BEKK(1,1) model or a conditional constant correlation GARCH(p,q) model. For each period the conditional covariance matrix is forecasted by the variance equation of the model.
The returns of the 15 countries are modelled and forecasted independently in 3 groups. The volatility forecast of the 3 models are combined in a block-diagonal matrix.
The filled GARCH block portfolio uses the same variance forecasts by block as the "block-GARCH" model. But now the volatility forecast of the 3 models are combined by the block diagonal matrix given from the "block-GARCH" model and the off-diagonal block elements are the appropriate blocks of the historical variance matrix of the buy-and-hold portfolio.

5 Portfolio evaluation

Back-testing is important to compare the results of different portfolio strategies with the benchmark returns (the MSCI Europe index). In our portfolio evaluation we use the following criteria:

1. The cumulative return (R_t)
   is calculated as $\sum_{t=1}^{T} r_t$, where r_t are the monthly portfolio returns and T goes from 1 year up to $T = 31$ months.

2. The standard deviation (SD)

of the returns r_t over a period of length T is defined as

$$SD = \sigma(r_t) = \sqrt{(\frac{1}{T}\sum_{t=1}^{T}(r_t - \hat{r}))} \tag{8}$$

The annualized (standard deviation (ann.SD) is obtained by multiplying the standard deviation of the portfolio returns by $\sqrt{12}$.

3. The *Sharpe* ratio
   for a period T is defined as the expected excess return of the portfolio divided by the SD of the portfolio. We compute the *Sharpe* ratio [1] as the ratio of the average return $ave(return)$ and the SD of the returns for a certain period:

$$Sharpe(return) = ave(return)/\sigma_{ave}(return). \tag{9}$$

   In similar way the (realized) *Sharpe* ratio with respect to the geometric mean can be defined:

$$Sharpe(return) = geom(return)/\sigma_{geom}(return). \tag{10}$$

4. The *Success* rate (S_T)
   for a period T is defined as the percentage of times in which the portfolio returns are larger than the benchmark returns.

$$S_T = \frac{1}{T}\sum_{t=1}^{T} I_t \tag{11}$$

   where I_t is a "success" indicator and is defined as

$$I_t = \left\{ \begin{array}{ll} 0 & \text{if} \quad r_t \leq r_{benchmark}; \\ 1 & \text{if} \quad r_t > r_{benchmark}. \end{array} \right\}$$

[1]Although the *Sharpe* ratio is the ratio of the expected **excess** return and the SD, many investment funds use the cumulative returns for given period divided by the SD ratio in this period as measure for the (realized) *Sharpe* ratio (see e.g. the Frankfurter Allgemeine Zeitung from 25 September 2000).

5. The standard deviation of the tracking error $\sigma(TE)$
for a period T is computed as the annualized SD of the differences of the portfolio returns and the benchmark returns:

$$\sigma(TE) = \sigma(r_t - r_{benchmark}). \tag{12}$$

6. The *Information* ratio IR
for a period T is defined as the cumulative active return divided by active standard deviation (measured by the SD of the tracking error):

$$IR_T = cum_T(r_t - r_{benchmark})/\sigma_T(TE), \tag{13}$$

where $cum_T(r_t - r_{benchmark})$ stands for the cumulative active return for period T.

7. The geometric mean.
The logarithm of the geometric mean $\bar{r}_{lg}$ is defined as the mean of the log growth rates, i.e.

$$\bar{r}_{lg} = log(1 + \bar{r}_{geom}) = \frac{1}{T}\sum_{t=1}^{T} log(1 + r_t) \tag{14}$$

where $1 + r_t$ are the growth rates of the returns r_t. The geometric mean is then obtained as $\bar{r}_{geom} = e^{r_{lg}} - 1$.

8. The standard deviation of the geometric mean SD_{geom}.
This risk measure is defined in analogy to the ordinary standard deviation: The standard deviation from the geometric mean $\bar{x}_{geom}$ is

$$SD_{geom} = \frac{1}{T}\sum_{t=1}^{T}(x_t - \bar{x}_{geom})^2 \tag{15}$$

Table 2 summarizes the results according to the above criteria. The results show large differences for the cumulative returns over the 12-31 months of the evaluation period. The factor portfolio FP and the global European GE

	GARCH portfolio	**block GARCH portfolio**	**filled GARCH blocks p.**	**MSCI Europe returns**
cum. Return	22.65%	12.89%	16.23%	-16.07%
ann. Return	9.06%	5.15%	6.4%	-6.43%
ann. SD	12.50	11.77	19.31	15.37
$Sharpe.ratio_{ave}$	.72	.43	0.34	-.41
$ann.Geom$	8.60%	4.53%	4.78%	-7.30%
$ann.SD_{geom}$	12.51	11.78	19.32	13.4%
$Sharpe.ratio_{geom}$	.69	.38	0.25	-.54
$geom.mean_{mo}$	0.69%	0.37%	0.39%	-0.63%
$geom.SD_{mo}$	3.61	3.40	5.58	3.87
Success rate	70%	60%	60%	
$\sigma(Tracking - err.)$	22.04	14.41	23.79	
$Inform.ratio_{cum}$	1.75	2.0	1.35	

Table 2: Performance from January 1999 until January 2001: Portfolio returns of the full GARCH, the diagonal block GARCH, the filled block GARCH model and the MSCI Europe return (benchmark).

portfolio, both are based on the forecasted variance matrix by multivariate GARCH models, yields about 8% more returns than the benchmark in the evaluation period. The *Success* rate is larger than 50% and the *Sharpe* ratio is more than 3 times larger than the *Sharpe* ratio of the MSCI Europe index. Interestingly, the "buy-and-hold" portfolio which uses a constant (average) variance structure leads to very bad portfolio performance and the only negative information ratio. Therefore the portfolio has a negative *Sharpe* ratio and the return lies 28% below the benchmark. Figure 5 plots the cumulative returns of the Global European portfolio and the MSCI Europe index.
This result shows that assuming a constant volatility model (for the period 1999-2000 for 15 European countries lead to bad volatility forecasts and therefore a bad performance of the buy-and-hold portfolio. When blocks of countries are used and forecasts are obtained for each block, then this

	Factor portfolio	**GE portfolio**	**"B&H" portfolio**	**MSCI Europe**
cum. Returns	12.25%	11.19%	-25.81%	3.07%
ann. Returns	5.88%	5.37%	-12.4%	1.47%
ann. SD	17.9	14.6	18.6	13.4
Sharpe ratio	0.33	0.36	-0.67	0.11
Success rate	60%	56%	44%	
$\sigma(Tracking - error)$	15.53	9.41	21.85	
$Inform.ratio_{cum}$	0.59	0.86	-1.32	

Table 3: Performance from January 1999 until January 2001: Returns from the factor (FP), global European (GE), buy-and-hold (B&H) portfolio and MSCI Europe.

compound volatility structure by 3 multivariate GARCH models lead to a better portfolio return than a buy-and-hold portfolio (with a simple constant covariance structure). The best portfolio yield is obtained if we predict the complete volatility structure for all European stock indices. This means that volatilities between European countries are driven by a complex dynamic interaction structure in the second moments. These interactions are surprisingly high-dimensional and cannot be reduced easily to a simpler model.

Furthermore, approximations of the full model by country blocks which are formed according to economic or geographical principles are not a satisfying solution either, since such a model cannot cover all the important dynamic variance interactions. If we assume independence between the blocks we find a worse portfolio performance than replacing the missing covariances by a constant (historical) covariance structure. This shows that conditionally the volatility structure is changing from period to period quite actively; if these changes can be predicted successfully then we obtain better portfolio results.

From Table 3 we see that the factor portfolio has a slightly higher return than the GE portfolio, but the *Sharpe* ratio of the factor portfolio is smaller because of the larger standard deviation. Figure 5 shows the cumulative returns of the portfolios in Table 3. Note that the buy-and-hold portfolio started to become quite worse with the beginning of the year 2000. This shows that the GE and the factor portfolio could adapt to a changing volatility structure while the buy-and-hold portfolio which is based on historical variances could not follow the new volatility structure.

6 Conclusions

This paper has demonstrated that the quantitative volatility timing for portfolio management is quite successful and can give higher returns than the benchmark. The weights are determined by mean-variance optimization and are quite different from the benchmark weights can be negative. Therefore we allow short sales in our portfolio because optimized weights based on volatility predictions are in some cases negative. While we have not included trading costs in our analysis we can include them easily by a proportional percentage fee but this will not change our results, i.e. the fact that outperformance can be obtained from our volatility timing strategy. As we have shown in PP ([**?**]) the trading cost will reduce the gross return by about 2% points.

It is interesting to see that volatility forecasts and country grouping by techniques like factor analysis can outperform the benchmark and the buy-and-hold strategy. Since the best performance of a European portfolio is obtained by a 15-dimensional volatility model, we conclude that the volatility interaction between all European indices is very active and no simplification of the volatility structure is possible without considerable loss of the portfolio return. No volatility timing model could be found from simpler and lower-dimensional models that produces portfolio yields which are close to the best European model we have found. We conclude that this is due to a conditionally changing volatility structure where all European countries are involved. Even small countries like Luxembourg (the Greece index was too short to be included) are contributing to a better overall portfolio return.

Further research will show if new methods allow a better insight into the changing volatility structure of Europe (e.g. using dynamic macro-economic models) and what features of multivariate volatility models are responsible for better results of volatility timing. We conclude that volatility timing strategies by high-dimensional volatility models for European countries can improve the performance of quantitative portfolios considerably.

7 Appendix: The multivariate GARCH (or BEKK) Model

Let $r_t = (r_t^1, ..., r_t^N)'$ be a N dimensional vector of returns at time t and we specify the following multivariate GARCH model

$$r_t = \mu + \epsilon_t, \quad t = 1, ..., T, \tag{16}$$

with

$$\epsilon_t | I_{t-1} \sim N(0, H_t) \tag{17}$$

where μ is a constant mean vector of dimension N and the heteroskedastic errors ϵ_t are *conditionally* multivariate normally distributed. Each element of H_t depends on p lagged values of squares and cross-products of $\epsilon_t^l, l = 1, ..., N$ and on q lagged values of H_t.

Defining $h_t = vechH_t$ as the vectorisation of a symmetric matrix and $\eta_t = vec(\epsilon_t \epsilon_t')$ then the multivariate $GARCH(p,q)$ parameterization of the variance matrix can be written as

$$h_t = a_0 + A_1 \eta_{t-1} + ... + A_p \eta_{t-p} + B_1 h_{t-1} + ... + B_q h_{t-q} \tag{18}$$

where a_0 is a $n \times 1$ vector with $n = N(N+1)/2$ and the A_i's and B_i's are $n \times n$ parameter matrices. This parameterization is also called *vec* representation. Bollerslev et al. [**?**] have proposed a *diagonal* representation, in which each element of the variance matrix $h_{jk,t}$ depends only on past variances and the past values of $\epsilon_t^l \epsilon_t^k$. This means that the conditional variances depend on past own variances and past squared residuals; likewise the covariances depend on past own covariances and cross products of residuals. In the

vec representation the *diagonal* model is obtained by assuming a diagonal structure of the matrices A_i and B_i.
In both representations it is difficult to impose the condition of a positive definite variance matrix for the estimation procedure. Engle and Kroner [?] propose the so-called BEKK representation which ensures the condition of a positive definite conditional variance matrix by a special matrix form. This BEKK representation parameterizes the variance matrix by the following way:

$$H_t = A_0 A_0' + \sum_{i=1}^{p} A_i(\epsilon_{t-i}\epsilon_{t-i}')A_i' + \sum_{i=1}^{q} B_i H_{t-i} B_i'. \tag{19}$$

Thus, the variance matrix forecast is obtained from

$$\hat{H}_{t+1} = \hat{A}_0 \hat{A}_0' + \sum_{i=1}^{p} \hat{A}_i E_t(\epsilon_{t+1-i}\epsilon_{t+1-i}')\hat{A}_i' + \sum_{i=1}^{q} \hat{B}_i H_{t+1-i} \hat{B'}_i, \tag{20}$$

where E_t is the conditional expectation operator.

7.1 Factor analysis

Factor analysis is a mathematical method which can be used to explain the correlation matrix between a large set of variables by a small number of underlying factors which are not directly observable.
Denote by x a p-dimensional return vector with mean μ and variance matrix H, then the k-factor model can be written in the form

$$x = \mu + \Lambda f + \epsilon \tag{21}$$

where $\Lambda = \{\lambda_{ij}\}, i = 1, \ldots, p, j = 1, \ldots, k$, is a matrix of rank $k \leq p$ called the matrix of factor loadings and f and ϵ are random vectors with mean zero and covariance matrix I_k and ψ, respectively. They are interpreted as the k underlying *common* factors f and the p *unique* factors ϵ, i.e. the residuals associated with the original variable x. Assuming normality for the error term, the factor model is given by

$$x \sim N[\mu + \Lambda f, H], \tag{22}$$

Thus, the variance matrix H can be decomposed into a factor variance matrix $\Lambda\Lambda'$ and an error variance matrix $\Psi = Var(\epsilon)$:

$$H = \Lambda\Lambda' + \Psi. \tag{23}$$

There are two main techniques for estimating the factors: the principal factor estimate and the maximum likelihood estimate (see Mardia et al. (1979, [?]). The solution to equation (23) is not unique (unless $k = 1$); if G is any $(k \times k)$ orthogonal transformation matrix with $G^{-1} = G'$, then

$$H = (\Lambda G')(G\Lambda') + \Psi \tag{24}$$

which has the form of equation (23) if $\Delta = \Lambda G$ denote the new matrix of rotated factor loadings. Thus, any solution of a factor analysis can be rotated to arrive at a new solution. There are many different criteria for choosing the appropriate rotation. We are using the varimax criterion (see Mardia et al. [?]).
We are applying factor analysis to data described in the section above. The loading matrix gives information about how strong each variable is connected ('loaded') to each factor. Figure 6 shows the 6 largest loadings for a 3 dimensional factor model. Looking at the factor loading we split the original data set into three blocks of 8, 5 and 3 countries: Belgium, France, Netherlands, Germany, Switzerland, Portugal and Denmark make up the first block. The second block contains Sweden, Spain, Finland, Italy and Norway. The third block are Great Britain, Ireland and Austria. The biplot graph is a graphical representation of the returns for the first 2 factors and shows the relationship between both, the original variables and the original data points (see figure 7).

MSCI Indices	Ireland .IE	Italy .IT	Portugal .PT	Spain .ES
Min	-0.1963	-0.2095	-0.1505	-0.2452
25% Quartile	-0.0274	-0.047964	-0.0384	-0.0314
Mean	0.0046	0.0042	0.0024	0.0065
Median	0.0074	0.0024	0.0037	0.0036
75% Quartile	0.0362	0.049661	0.0423	0.0564
Max	0.1668	0.1939	0.1944	0.1938
Std.Dev.	0.0555	0.0718	0.0633	0.0675
Skewness	0.1840	0.1032	0.0910	0.3957
Kurtosis	1.6502	0.1562	0.1699	1.3002

Table 4: Summary statistics of the monthly returns of the MSCI indices for Ireland, Italy, Portugal and Spain from January 1990 until January 2001 (133 observations).

MSCI Indices	Austria .AT	France .FR	Germany .DE	Suisse .CH	UK .GB
Min	-0.2666	-0.1381	-0.1962	-0.1710	-0.0981
25% Quar.	-0.0403	-0.0248	-0.0218	-0.0148	-0.0223
Mean	-0.0024	0.0077	0.0066	0.0109	0.0063
Median	0.0002	0.0103	0.0154	0.0090	0.0050
75% Quar.	0.0362	0.0385	0.0403	0.0426	0.0386
Max	0.1799	0.1223	0.1574	0.1524	0.1357
Std.Dev.	0.0674	0.0507	0.0564	0.0510	0.0444
Skewness	0.5379	0.2495	0.8610	0.5252	0.0056
Kurtosis	2.077	0.0584	1.8622	1.3374	-0.0270

Table 5: Summary statistics of the monthly returns of the MSCI indices for Austria, France, Germany, Switzerland and United Kingdom (Britain) from January 1990 until January 2001 (133 observations).

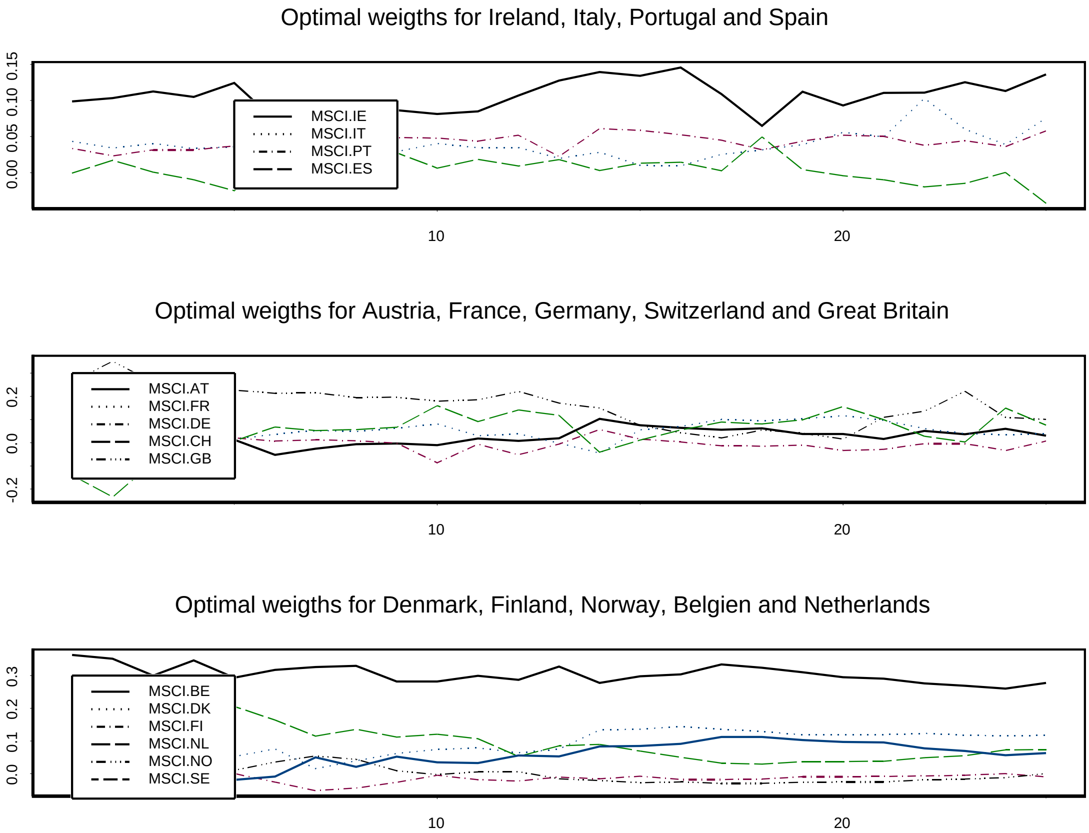

Figure 1: Three groups of country weights of the GE portfolio from January 1999 until January 2001. E.g., the weights for January 1999 are computed from equation (7) using the forecasted variance matrix of the VAR-GARCH model for January 1999. This procedure is repeated for consecutive 31 months.

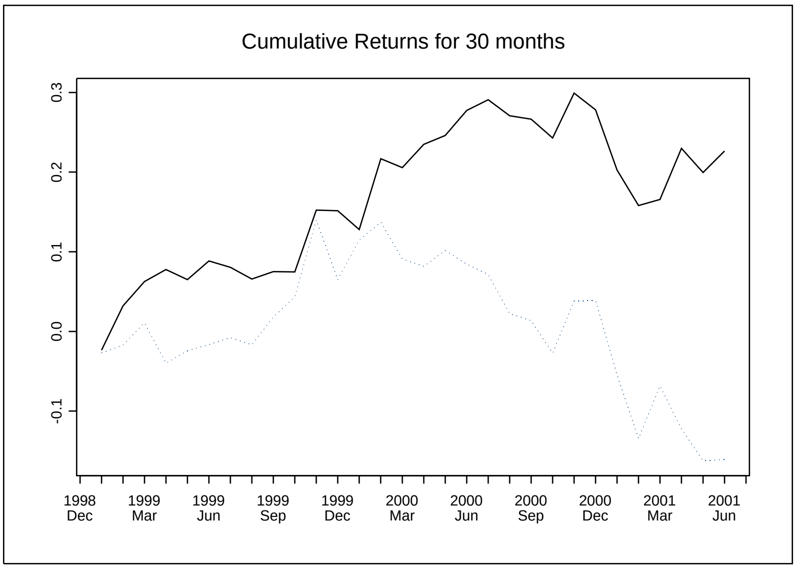

Figure 2: The figure shows the cumulative returns of the GARCH portfolio and the cumulative returns of the MSCI Europe index in the evaluation period (January 1999-June 2001).

MSCI Indices	**Belgium BE**	**Denmark DK**	**Finland FI**	**N.lands NL**	**Norway NO**	**Sweden SE**
Min	-0.2084	-0.1200	-0.2182	-0.1327	-0.3270	-0.2434
25% Quar.	-0.0215	-0.0292	-0.0474	-0.0102	-0.0477	-0.0316
Mean	0.0044	0.0076	0.0153	0.0092	0.0021	0.0102
Median	0.0082	0.0120	0.0051	0.0102	0.0044	0.0091
75% Quar.	0.0305	0.0397	0.0759	0.0403	0.0500	0.0586
Max	0.1057	0.1241	0.2657	0.0959	0.1537	0.2054
Std.Dev.	0.0458	0.0512	0.0893	0.0438	0.0683	0.0708
Skewness	0.7739	0.1188	0.0357	0.7006	0.7911	0.2742
Kurtosis	2.8863	-0.3239	0.1825	0.8203	2.9664	0.8349

Table 6: Summary statistics of the monthly returns of the MSCI indices for Belgium, Denmark, Finland, Netherlands, Norway and Sweden from January 1990 until January 2001 (133 observations).

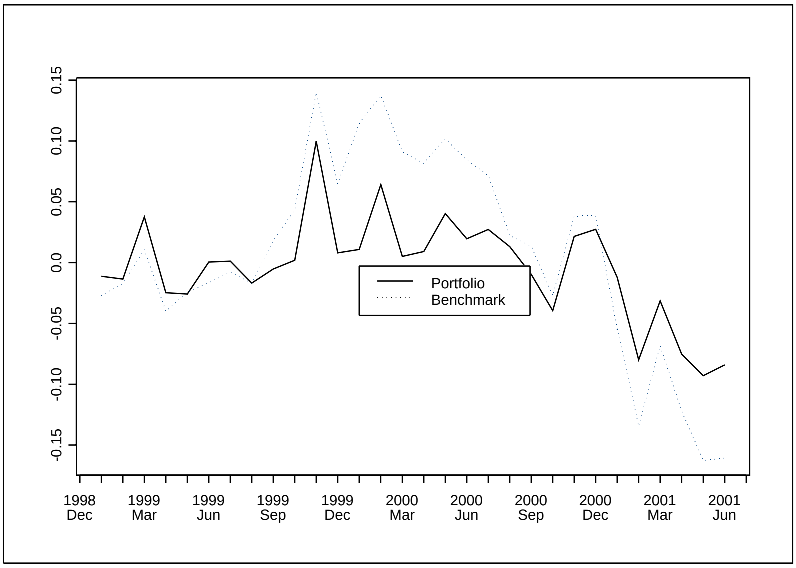

Figure 3: The figure shows the cumulative returns of the '3 block GARCH' portfolio (block-diagonal matrix) and the cumulative returns of the MSCI Europe index in the evaluation period (January 1999-June 2001).

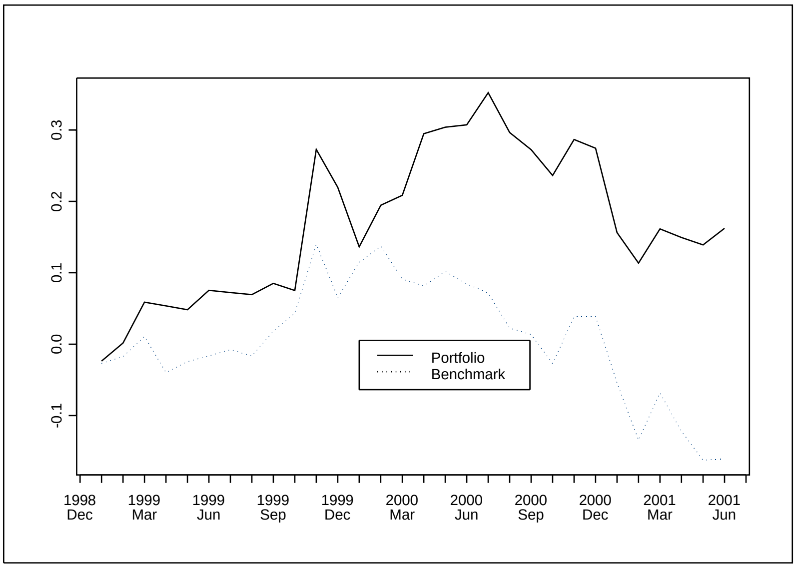

Figure 4: The figure shows the cumulative returns of the 'block&fill GARCH' portfolio (using the full correlation matrix) and the cumulative returns of the MSCI Europe index in the evaluation period (January 1999-June 2001).

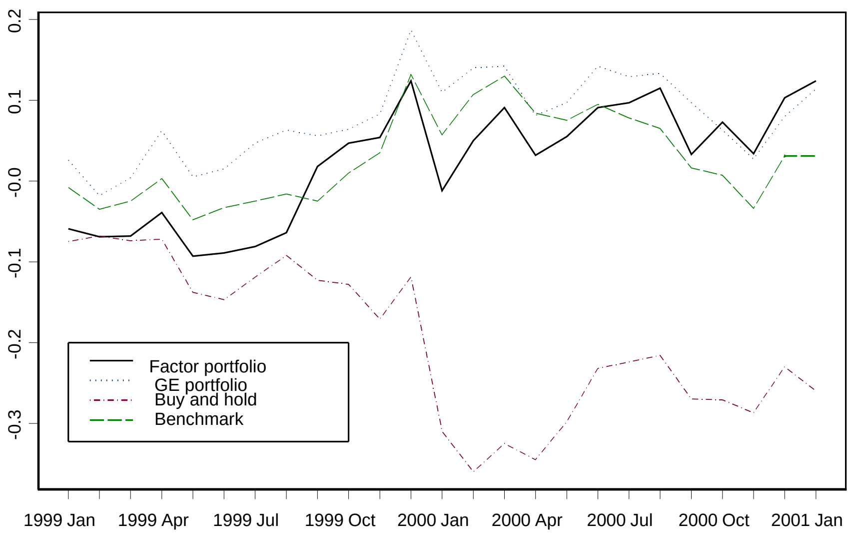

Figure 5: The figure plots the cumulative returns of the GMV portfolio and the cumulative returns of the MSCI Europe index in the evaluation period (January 1999-January 2001).

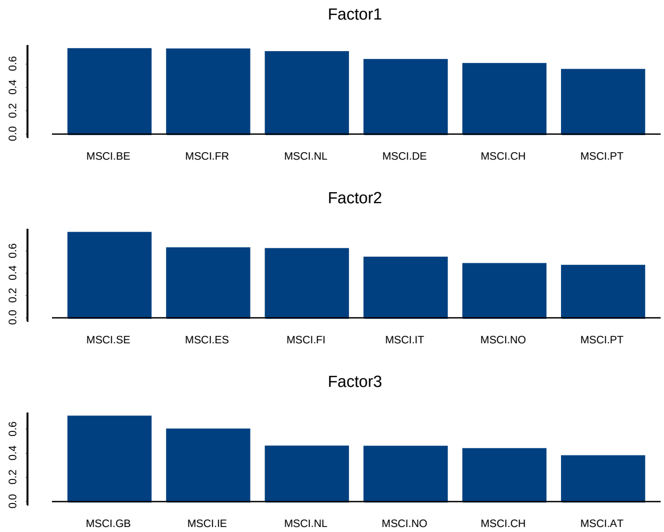

Figure 6: Factor loadings

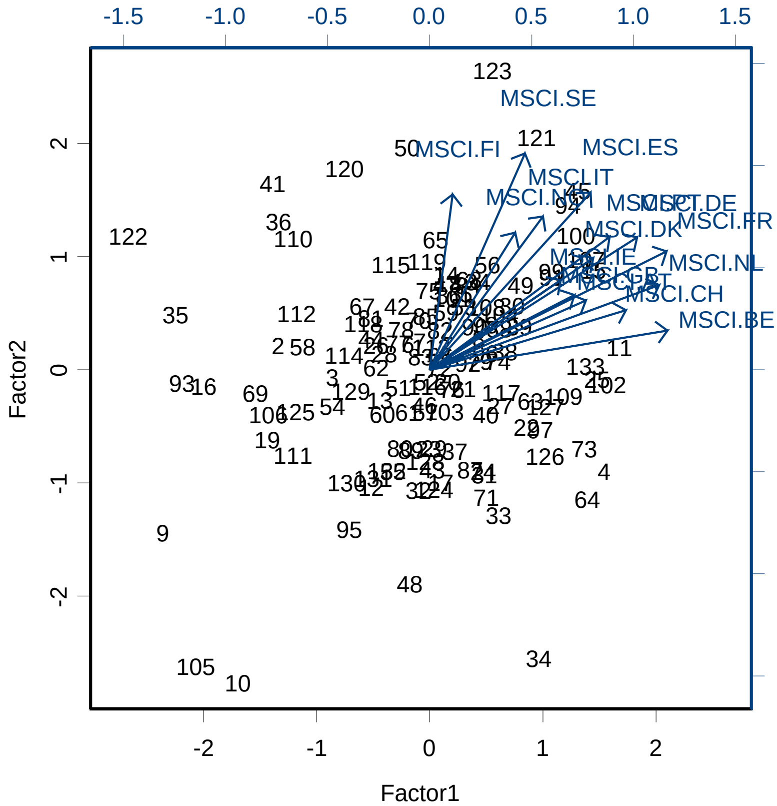

Figure 7: Biplot for the 3 factor solution: All observations (numbers) and the variables (arrows) are shown. Countries which are close together are used to form a group.