

Time-Varying Uncertainty and the Credit Channel*

Abstract

We extend the Carlstrom and Fuerst (1997) agency cost model of business cycles by including time varying uncertainty in the technology shocks that affect capital production. We first demonstrate that standard linearization methods can be used to solve the model yet second moment effects still influence equilibrium characteristics. The effects of the persistence of uncertainty are then analyzed. Our primary findings fall into four categories. First, it is demonstrated that uncertainty affects the level of the steady-state of the economy so that welfare analyses of uncertainty that focus entirely on the variability of output (or consumption) will understate the true costs of uncertainty. A second key result is that time varying uncertainty results in countercyclical bankruptcy rates - a finding which is consistent with the data and opposite the result in Carlstrom and Fuerst. Third, we show that persistence of uncertainty affects both quantitatively and qualitatively the behavior of the economy. Finally, we demonstrate that the magnitude of changes in uncertainty affecting the economy could be quite large; the implication is that second moment effects may be an important determinant of macroeconomic behavior.

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1 Introduction

While technology has quickened production adjustments, human nature remains unaltered. We respond to a heightened pace of change and its associated uncertainty in the same way we always have. We withdraw from action, postpone decisions, and generally hunker down until a renewed, more comprehensible basis for acting emerges. In its extreme manifestation, many economic decision makers not only become risk averse but attempt to disengage from all risk.¹

The last ten years has seen a proliferation of macroeconomic models that highlight the role of financial intermediaries in business cycle activity. With variations on this theme referred to as models of the credit channel, agency cost models, or financial accelerator models, the common element is that lending activity is characterized by asymmetric information between borrowers and lenders. As a consequence, interest rates may not move to clear lending markets (as in models with moral hazard and adverse selection elements) or firms' net worth may play a critical role as collateral in influencing lending activity (as in models with agency costs). While debate on the empirical support for these models continues, there is little doubt that, as a whole, they have improved our understanding of financial intermediation and broadened the scope of how monetary policy, through the impact of interest rates on firms' net worth, can influence macroeconomic performance.²

With the central role that information plays in these models, they present a potentially rich environment to study the effects that changes in uncertainty have on aggregate economic behavior. Of course, Chairman Greenspan, as reflected in the quote above, is not the first to note that changes in economic uncertainty and, in particular, variations in the confidence one attaches to forecasts, surely affect household savings decisions as well as firms' investment plans. For example, Bernanke (1983), Abel (1983), Pindyck (1991) and Gilchrist and Williams (2000) all explore the role that uncertainty has on investment decisions. In this paper, our goal is to continue this

¹ Testimony of Chairman Alan Greenspan before the Committee on Banking, Housing, and Urban Affairs, U.S. Senate. February 13, 2001

² The credit channel literature is large and continues to expand. Some prominent contributions are: Williamson (1987), Bernanke and Gertler (1989, 1990), Bernanke, Gertler, and Gilchrist (1999), Kiyataki and Moore (1997), and Carlstrom and Fuerst (1997). Walsh (1998) presents an overview, both theoretical and empirical, of the literature.

exploration but to do so in an environment that also models lending activity. To do this, we introduce time-varying uncertainty, i.e. second moment effects, into the agency cost model of Carlstrom and Fuerst (1997). This model is particularly attractive for our purposes since it incorporates a lending channel (for investment) that is characterized by asymmetric information between lenders and borrowers. In addition, the economic environment is a variant of a typical real business cycle model so that key parameters can be calibrated to the data. Within this setting, we model time varying uncertainty as a mean preserving spread in the distribution of the technology shocks affecting capital production and explore how changes in uncertainty affect equilibrium characteristics.

We first demonstrate that linearization solution methods can be employed yet this does not eliminate the effects of second moments. That is, in solving for the linear equilibrium policy functions, the vector of state variables includes the variance of technology shocks buffeting the capital production sector. While Sims (2001), Collard and Juillard (2001), and Schmitt-Grohe and Uribe (2003) have developed more general solution methods that also permit the analysis of second moment effects, the straightforward extension of linearization methods employed here has appeal due to tractability.

The analysis of the equilibrium characteristics presents some interesting features. One of the primary findings is that time varying uncertainty produces countercyclical bankruptcy rates. In contrast, Carlstrom and Fuerst's (1997) analysis of aggregate technology shocks produced the counterfactual prediction of procyclical bankruptcy rates. Another prediction is that increases in uncertainty will, through the lending mechanism, affect the expected return to capital (that is, exogenous second moment effects have endogenous first moment implications); this will, in turn, result in greater consumption but a reduction in investment. Hence, the analysis presented here demonstrates that second moment effects, not surprisingly, expand the set of equilibrium characteristics; moreover, in some instances, first and second moment effects move in opposite directions. This may have important consequences for understanding historical business cycles.

That is, historical business cycles can be differentiated by whether the shocks are predominantly to aggregate supply or aggregate demand. Our analysis suggests that another useful distinction may be the role of information; namely, is it the first or second moments of the shocks hitting the economy that is dominant in influencing equilibrium characteristics.

These qualitative results are then examined quantitatively; specifically, we attempt to measure changes in uncertainty by using forecast data from the Professional Forecasters Survey. Our analysis, while preliminary, suggests that the magnitude of second moment shocks could be quite large relative to aggregate technology shocks. Hence, empirical analyses of models of the broad credit channel that do not account for time-varying uncertainty may be understate the importance of the financial intermediation sector in the economy.

Our analysis is closely related to a recent paper by Richard Cooper and João Ejarque (2000) in which they examine the business cycle characteristics of an economy subject to shocks that impact the financial intermediation sector. Their model presents a linear technology that transforms deposits into loans and this technology is subject to stochastic productivity shocks. An analysis of the equilibrium behavior of the model economy leads Cooper and Ejarque to reach a broadly negative stance on the role that financial sector shocks play in the actual economy. In particular, the basic model implies that labor and investment are countercyclical and negatively correlated with consumption. In addition, the capital stock is predicted, counterfactually, to be more volatile than output. We see our analysis as a refinement of the Cooper and Ejarque analysis in that the shocks to the financial sector are explicitly modeled (i.e. second moment shocks versus financial sector productivity shocks); moreover, the relatively rich specification of the lending channel in the Carlstrom and Fuerst (1997) economy permits an analysis of the endogenous response to these information shocks. Because of these endogenous responses, the credit channel model displays behavior more in line with actual economies: investment and labor are procyclical and capital, while more volatile than in an economy subject only to productivity shocks (as in a standard real business cycle model), is not wildly out of line with the data. We do find that consumption in the

model is countercyclical, but this behavior is similar to models like Greenwood, Hercowitz, and Krusell (1997) in which the economy is subject to investment-specific technology shocks. While the mechanism here is somewhat different, the conclusion is similar in that the shocks in our economy, like theirs, affects the price of investment relative to consumption goods.

The next section presents the model while the following section discusses equilibrium characteristics. The final section offers some concluding comments.

2 Model

We employ the agency cost business cycle model of Carlstrom and Fuerst (1997) to address the financial intermediaries' role in the propagation of productivity shocks and extend their analysis by introducing time-varying uncertainty. Since, for the most part, the model is identical to that in Carlstrom and Fuerst, the exposition of the model will be brief.

The model is a variant of a standard RBC model in which an additional production sector is added. This sector produces capital using a technology which transforms investment into capital. In a standard RBC framework, this conversion is always one-to-one; in the Carlstrom and Fuerst framework, the production technology is subject to technology shocks. (The aggregate production technology is also subject to technology shocks as is standard.) This capital production sector is owned by entrepreneurs who finance their production via loans from a risk neutral financial intermediation sector - this lending channel is characterized by a loan contract with a fixed interest rate. (Both capital production and the loans are intra-period.) If a capital producing firm realizes a low technology shock, it will declare bankruptcy and the financial intermediary will take over production; this activity is subject to monitoring costs. With this brief description, we now turn to an explicit characterization of the economy.

2.1 Households

The representative household is infinitely lived and has expected utility over consumption c_t and leisure $1 - l_t$ with functional form given by:

$$E_0 \mathbb{P}_{t=0} \beta^t [\ln(c_t) + \nu(1 - l_t)] \quad (1)$$

where E_0 denotes the conditional expectation operator on time zero information, $\beta \in (0, 1)$, $\nu > 0$, and l_t is time t labor. The household supplies labor, l_t , and rents its accumulated capital stock, k_t , to firms at the market clearing real wage, w_t , and rental rate r_t , respectively, thus earning a total income of $w_t l_t + r_t k_t$. The household then purchases consumption good from firms at price of one (i.e. consumption is the numeraire), and purchases new capital, i_t , at a price of q_t . Consequently, the household's budget constraint is

$$w_t l_t + r_t k_t \geq c_t + q_t i_t \quad (2)$$

The law of motion for households' capital stock is standard:

$$k_{t+1} = (1 - \delta) k_t + i_t \quad (3)$$

where $\delta \in (0, 1)$ is the depreciation rate on capital.

The necessary conditions associated with the maximization problem include the standard labor-leisure condition and the intertemporal efficiency condition associated with investment. Given the functional form for preferences, these are:

$$\nu c_t = w_t \quad (4)$$

$$\frac{q_t}{c_t} = \beta E_t \frac{q_{t+1} (1 - \delta) + r_{t+1}}{c_{t+1}} \quad (5)$$

2.2 Firms

The economy's output of the consumption good is produced by firms using Cobb-Douglas technology³

$$Y_t = \theta_t K_t^{\alpha_K} H_t^{\alpha_H} (H_t^e)^{\alpha_{H^e}} \quad (6)$$

where Y_t represents the aggregate output, θ_t denotes the aggregate technology shock, K_t denotes the aggregate capital stock, H_t denotes the aggregate household labor supply, H_t^e denotes the aggregate supply of entrepreneurial labor, and $\alpha_K + \alpha_H + \alpha_{H^e} = 1$.⁴

The profit maximizing representative firm's first order conditions are given by the factor market's condition that wage and rental rates are equal to their respective marginal productivities:

$$w_t = \theta_t \alpha_H \frac{Y_t}{H_t} \quad (7)$$

$$r_t = \theta_t \alpha_K \frac{Y_t}{K_t} \quad (8)$$

$$w_t^e = \theta_t \alpha_{H^e} \frac{Y_t}{H_t^e} \quad (9)$$

where w_t^e denotes the wage rate for entrepreneurial labor.

2.3 Entrepreneurs

A risk neutral representative entrepreneur's course of action is as follows. To finance his project at period t , he borrows resources from the Capital Mutual Fund according to an optimal financial contract. The entire borrowed resources, along with his total net worth at period t , are then

³ Note that we denote aggregate variables with upper case while lower case represents per-capita values. Prices are also lower case.

⁴ As in Carlstrom and Fuerst, we assume that the entrepreneur's labor share is small, in particular, $\alpha_{H^e} = 0.0001$. The inclusion of entrepreneurs' labor into the aggregate production function serves as a technical device so that entrepreneurs' net worth is always positive, even when insolvent.

invested into his capital creation project. If the representative entrepreneur is solvent after observing his own technology shock, he then makes his consumption decision; otherwise, he declares bankruptcy and production is monitored (at a cost) by the Capital Mutual Fund.

2.4 Optimal Financial Contract

The optimal financial contract between entrepreneur and the Capital Mutual Fund is described by Carlstrom and Fuerst (1997). But for expository purposes as well as to explain our approach in addressing the second moment effect on equilibrium conditions, we briefly outline the model.

The entrepreneur has access to a stochastic technology that transforms i_t units of consumption into $\omega_t i_t$ units of capital. In Carlstrom and Fuerst (1997), the technology shock ω_t was assumed to be distributed as *i.i.d.* with $E(\omega_t) = 1$. While we maintain the assumption of constant mean, we assume that the standard deviation is time-varying. Specifically, we assume that the standard deviation of the capital production technology shock is governed by the following AR(1) process

$$\sigma_{\omega,t} = \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t-1}^{\zeta} \mu_t \quad (10)$$

where $\zeta \in (0,1)$ and $\mu_t \sim i.i.d$ with a mean of unity.⁵ The unconditional mean of the standard deviation is given by $\bar{\sigma}_{\omega}$. The realization of ω_t is privately observed by entrepreneur – banks can observe the realization at a cost of μi_t units of consumption.

The entrepreneur enters period t with one unit of labor endowment and z_t units of capital. Labor is supplied inelastically while capital is rented to firms, hence income in the period is $w_t + r_t z_t$. This income along with remaining capital determines net worth (denominated in units of consumption) at time t :

$$n_t = w_t + z_t (r_t + q_t (1 - \delta)) \quad (11)$$

⁵ This autoregressive process is used so that, when the model is log-linearized, $\hat{\sigma}_{\omega,t}$ (defined as the percentage deviations from $\bar{\sigma}_{\omega}$) follows a standard, mean-zero AR(1) process.

With a positive net worth, the entrepreneur borrows $(i_t - n_t)$ consumption goods and agrees to pay back $1 + r^k$ $(i_t - n_t)$ capital goods to the lender, where r^k is the interest rate on loans. Thus, the entrepreneur defaults on the loan if his realization of output is less than the re-payment, i.e.

$$\omega_t < \frac{1 + r^k (i_t - n_t)}{i_t} \equiv \bar{\omega}_t \quad (12)$$

The optimal borrowing contract is given by the pair $(i, \bar{\omega})$ that maximizes entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur). Denoting the *c.d.f.* and *p.d.f.* of ω_t as $\Phi(\omega_t)$ and $\phi(\omega_t)$ respectively, the contract is determined by the solution to:⁶

$$\max_{\{i, \bar{\omega}\}} qif(\bar{\omega}) \text{ subject to } qig(\bar{\omega}) \geq (i - n)$$

where

$$f(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega \phi(\omega) d\omega - [1 - \Phi(\bar{\omega})] \bar{\omega}$$

which can be interpreted as the fraction of the expected net capital output received by the entrepreneur⁷,

$$g(\bar{\omega}) = \int_0^{\bar{\omega}} \omega \phi(\omega) d\omega + [1 - \Phi(\bar{\omega})] \bar{\omega} - \Phi(\bar{\omega}) \mu$$

which represents the lender's fraction of expected capital output, $\Phi(\bar{\omega})$ is the bankruptcy rate so that $\Phi(\bar{\omega}) \mu$ denotes monitoring costs. Also note that $f(\bar{\omega}) + g(\bar{\omega}) = 1 - \Phi(\bar{\omega}) \mu$: the RHS is the average amount of capital that is produced – this is split between entrepreneurs and lenders.

Hence the presence of monitoring costs reduces net capital production.⁸

⁶ This notation is imprecise in that it implies the distributions are time-invariant. That is, the *c.d.f.* should be expressed as $\Phi_t(\omega_t) \equiv \Phi(\omega_t; \omega_{t-1}, \mu_t)$ with the *p.d.f.* expressed as $\phi_t(\omega_t) \equiv \phi(\omega_t; \omega_{t-1}, \mu_t)$. For simplicity, we suppress the time-notation.

⁷ The derivative of this function is $f'(\bar{\omega}) = \Phi(\bar{\omega}) - 1$. Thus, as $\Phi(\bar{\omega}) \in [0, 1]$, we have $f'(\bar{\omega}) \leq 0$. That is, as the lower bound for the realization of the technology shock (or the cutoff bankruptcy rate) increases, the entrepreneur's output share goes down.

⁸ This suggests that monitoring costs are akin to investment adjustment costs - in fact, Carlstrom and Fuerst demonstrate that this is the case. The important difference between this model and a model with adjustment costs is that entrepreneurs' net worth is an endogenous state variable that affects the dynamics of the economy - this feature is not present in an adjustment cost model.

The necessary conditions for the optimal contract problem are

$$\begin{aligned}
\frac{\partial(\cdot)}{\partial\bar{\omega}} &: qif'(\bar{\omega}) = -\lambda ig'(\bar{\omega}) \\
\Rightarrow \lambda &= \frac{-f^0(\bar{\omega})}{g'(\bar{\omega})} \\
\lambda &= \frac{f'(\bar{\omega})}{\phi(\bar{\omega})\mu + f'(\bar{\omega})} \\
\lambda &= \frac{1 - \Phi(\bar{\omega})}{1 - \Phi(\bar{\omega}) - \phi(\bar{\omega})\mu}
\end{aligned}$$

where λ is the shadow price of capital. This can be rewritten as:

$$1 - \frac{1}{\lambda} = \frac{\phi(\bar{\omega})}{1 - \Phi(\bar{\omega})}\mu \quad (13)$$

As shown by eq.(13), the shadow price of capital is an increasing function of the relevant Inverse Mill's ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals zero, then the shadow price equals the cost of capital production, i.e. $\lambda = 1$.

The second necessary condition is:

$$\frac{\partial(\cdot)}{\partial i} : qf(\bar{\omega}) = -\lambda [1 - qg(\bar{\omega})]$$

Solving for q using the first order conditions, we have

$$\begin{aligned}
q &= (f(\bar{\omega}) + g(\bar{\omega})) + \frac{\phi(\bar{\omega})\mu f(\bar{\omega})}{f'(\bar{\omega})} s^{-1} \\
&= 1 - \Phi(\bar{\omega})\mu + \frac{\phi(\bar{\omega})\mu f(\bar{\omega})}{f'(\bar{\omega})} s^{-1} \\
&\equiv [1 - D(\bar{\omega})]^{-1}
\end{aligned} \quad (14)$$

where $D(\bar{\omega})$ can be thought of as the total default costs.

Equation (14) defines an implicit function $\bar{\omega}(q)$ that is increasing in q , or the price of capital that incorporates the expected bankruptcy costs. The price of capital, q , differs from unity due to the presence of the credit market friction. That is, to compensate for the bankruptcy (monitoring) costs, there must be a premium on the price of capital. And this premium is set by the amount of monitoring costs and the probability of bankruptcy. (Note that $f'(\bar{\omega}) = \Phi(\bar{\omega}) - 1 < 0$.)

Finally, the incentive compatibility constraint implies

$$i = \frac{1}{(1 - qg(\bar{\omega}))} n \quad (15)$$

Equation (15) implies that investment is linear in net worth and defines a function that represents the amount of consumption goods placed in to the capital technology: $i(q, n)$. The fact that the function is linear implies that the aggregate investment function is well defined.

2.5 Entrepreneur's Consumption Choice

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than the household. This is represented by following expected utility function:

$$E_0 \mathbb{P}_{t=0} (\beta\gamma)^t c_t^e \quad (16)$$

where c_t^e denotes entrepreneur's consumption at date t , and $\gamma \in (0, 1)$. This new parameter, γ , will be chosen so that it offsets the steady-state internal rate of return to entrepreneurs' investment.

At the end of the period, the entrepreneur finances consumption out of the returns from the investment project implying that the law of motion for the entrepreneur's capital stock is:

$$z_{t+1} = n_t \frac{f(\bar{\omega}_t)^{1/2}}{1 - q_t g(\bar{\omega}_t)^{3/4}} - \frac{c_t^e}{q_t} \quad (17)$$

Note that the expected return to internal fund is $\frac{q_t f(\bar{\omega}_t) i_t}{n_t}$; that is, the net worth of size n_t is leveraged into a project of size i_t , entrepreneurs keep the share of the capital produced and capital is priced at q_t consumption goods. Since these are intra-period loans, the opportunity cost is 1.⁹

Consequently, the representative entrepreneur maximizes his expected utility function in equation (16) over consumption and capital subject to the law of motion for capital, equation (17), and the definition of net worth given in equation (11). The resulting Euler equation is as follows:

$$q_t = \beta \gamma E_t \left[q_{t+1} (1 - \delta) + r_{t+1} \right]^{\frac{1}{2}} \cdot \frac{q_{t+1} f(\bar{\omega}_{t+1})}{(1 - q_{t+1} g(\bar{\omega}_{t+1}))}^{\frac{3}{4}}$$

2.6 Financial Intermediaries

The Capital Mutual Funds (CMFs) act as risk-neutral financial intermediaries who earn no profit and produce neither consumption nor capital goods. There is a clear role for the CMF in this economy since, through pooling, all aggregate uncertainty of capital production can be eliminated. The CMF receives capital from three sources: entrepreneurs sell undepreciated capital in advance of the loan, after the loan, the CMF receives the newly created capital through loan repayment and through monitoring of insolvent firms, and, finally, those entrepreneur's that are still solvent, sell some of their capital to the CMF to finance current period consumption. This capital is then sold at the price of q_t units of consumption to households for their investment plans.

2.7 Equilibrium

There are four markets: labor markets for households and entrepreneurs, goods markets for consumption and capital.

$$H_t = (1 - \eta) l_t \tag{18}$$

⁹ As noted above, we require in steady-state $1 = \gamma \frac{q_t f(\bar{\omega}_t)}{(1 - q_t g(\bar{\omega}_t))}$.

where η denotes the fraction of entrepreneurs in the economy.

$$H_t^e = \eta \quad (19)$$

$$C_t + I_t = Y_t \quad (20)$$

where $C_t = (1 - \eta) c_t + \eta c_t^e$ and $I_t = \eta i_t$.

$$K_{t+1} = (1 - \delta) K_t + I_t [1 - \Phi(\bar{\omega}) \mu] \quad (21)$$

A competitive equilibrium is defined by the decision rules for $\{K_{t+1}, Z_{t+1}, H_t, H_t^e, q_t, n_t, i_t, \bar{\omega}_t, c_t, c_t^e\}$ where these decision rules are stationary functions of $\{K_t, Z_t, \theta_t, \sigma_{\omega,t}\}$ and satisfy the following equations¹⁰

$$\nu c_t = \theta_t \alpha_H \frac{Y_t}{H_t} \quad (22)$$

$$\frac{q_t}{c_t} = \beta E_t \frac{1}{c_{t+1}} \left[q_{t+1} (1 - \delta) + \theta_{t+1} \alpha_K \frac{Y_{t+1}}{K_{t+1}} \right] \quad (23)$$

$$q_t = 1 - \Phi(\bar{\omega}_t) \mu + \frac{\phi(\bar{\omega}) \mu f(\bar{\omega}_t)^{\delta-1}}{f'(\bar{\omega}_t)} \quad (24)$$

$$i_t = \frac{1}{(1 - q_t g(\bar{\omega}_t))} n_t \quad (25)$$

$$q_t = \beta \gamma E_t \left[q_{t+1} (1 - \delta) + \theta_{t+1} \alpha_K \frac{Y_{t+1}}{K_{t+1}} \right] \frac{q_{t+1} f(\bar{\omega}_{t+1})}{(1 - q_{t+1} g(\bar{\omega}_{t+1}))} \quad (26)$$

$$n_t = \theta_t \alpha_H \frac{Y_t}{H_t^e} + z_t \left[q_t (1 - \delta) + \theta_t \alpha_K \frac{Y_t}{K_t} \right] \quad (27)$$

$$Z_{t+1} = \eta n_t \frac{f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} - \eta \frac{c_t^e}{q_t} \quad (28)$$

$$\theta_{t+1} = \theta_t^\rho \xi_{t+1} \text{ where } \xi_t \sim i.i.d. \text{ with } E(\xi_t) = 1 \quad (29)$$

$$\sigma_{\omega,t+1} = \bar{\sigma}_\omega^{1-\zeta} \sigma_{\omega,t}^\zeta \mu_{t+1} \text{ where } \mu_t \sim i.i.d. \text{ with } E(\mu_t) = 1 \quad (30)$$

¹⁰ A more thorough presentation of the equilibrium conditions are presented in the Appendix.

3 Equilibrium Characteristics

3.1 Steady-state analysis

While our focus is primarily on the cyclical behavior of the economy, an examination of the steady-state properties of the economy is useful for two reasons. First, by studying the interaction between uncertainty (i.e. the variance of the technology shock affecting the capital production sector) and the steady-state, the intuition for how time-varying uncertainty affects the cyclical characteristics of the economy is improved. Second, it is important to point out that changes in the second moment of technology shocks affect the level of the economy - most notably consumption and output. That is, since the cyclical analysis presented in the next section is characterized in terms of deviations from steady-state, the impact of changes in uncertainty on the level of economic activity is lost.¹¹

For this analysis, we use, to a large extent, the parameters employed in Carlstrom and Fuerst's (1997) analysis. Specifically, the following parameter values are used:

Table 1: Parameter Values

β	α	δ	μ
0.99	0.36	0.02	0.25

Agents discount factor, the depreciation rate and capital's share are fairly standard in RBC analysis. The remaining parameter, μ , represents the monitoring costs associated with bankruptcy. This value, as noted by Carlstrom and Fuerst (1997) is relatively prudent given estimates of bankruptcy costs (which range from 20% (Altman (1984) to 36% (Alderson and Betker (1995) of firm assets).

The remaining parameters, (σ, γ) , determine the steady-state bankruptcy rate (which we denote as br and is expressed in percentage terms) and the risk premium (denoted rp) associated

¹¹ This statement is in reference to Lucas's analysis of the cost of business cycles (Lucas (1987) in which the trend and cycle are treated as distinct. In contrast, our analysis demonstrates that the cyclical behavior of the economy has implications for the level of the steady-state. If one were using an endogenous growth model, cyclical behavior may well have implications for the trend.

with bank loans.¹² (Also, recall that γ is calibrated so that the rate of return to internal funds is equal to $\frac{1}{\gamma}$.)¹³ While Carlstrom and Fuerst found it useful to use the observed bankruptcy rate to determine σ , for our analysis we treat σ and br as exogenous and examine the steady state behavior of the economy under different scenarios. In particular we consider the following four economies:¹⁴

Table 2: Four Economies

Economy	σ	br (%)	γ
Economy I (<i>C&F</i>)	0.207	0.974	0.9474
Economy II	0.30	0.974	0.9538
Economy III	0.30	1.2	0.9458
Economy IV	0.35	1.8	0.9287

Hence Economy II departs from the Carlstrom and Fuerst economy by having greater uncertainty in the technology shock but holds the bankruptcy rate at the same level used by Carlstrom and Fuerst (note that this implies that the internal rate of return $\frac{1}{\gamma}$ to entrepreneurs falls). Economy III then permits the bankruptcy rate to increase by roughly a third. The final economy increases both the degree of technological uncertainty and the steady-state bankruptcy rate. To examine the effects of these changes, Table 3 reports the behavior of several key variables; for all but two variables, these are presented as percentage deviations from the values in the Carlstrom and Fuerst economy. The risk premium differential is reported as an absolute change while the minimum technology shock defined in the lending contract (i.e. $\bar{\omega}$) in the three modified economies is reported

¹² The equations defining the steady-state are presented in the Appendix. This derivation also demonstrates that the parameter η (the fraction of entrepreneurs in the economy) is strictly a normalization and does not influence equilibrium characteristics.

¹³ The fraction of entrepreneurs in the economy, η , is not a critical parameter for the behavior of the economy. As Carlstrom and Fuerst note, it is simply a normalization. Aggregate consumption in the model is indeed a weighted average of household and entrepreneurial consumption but the weights are determined by the steady-state level of per-capita consumption for these groups. This is endogenously determined - but not by η . This is demonstrated at the end of the Appendix.

¹⁴ In Table 2, the values of γ are reported strictly for comparison. That is, once the values of σ and br are specified, the value of γ is determined endogenously.

Table 3: Steady-state behavior

(comparison to Carlstrom & Fuerst Economy)

<i>variable</i>	Economy II	Economy III	Economy IV
<i>c</i>	-0.19	-0.44	-1.18
<i>k</i>	-0.51	-1.13	-3.05
<i>rp</i>	0.54	1.06	2.98
<i>q</i>	0.35	0.78	2.11
$\bar{\omega}^{15}$	0.47	0.49	0.45
<i>z</i>	28.4	25.0	28.4
<i>n</i>	28.7	25.7	30.4

Note that increases in uncertainty reduce the steady-state level of consumption and the aggregate capital stock monotonically. For the high variance, high bankruptcy rate economy (Economy IV), the reduction in steady-state consumption is greater than 1% - a non-trivial amount and similar in magnitude to welfare losses reported by Lucas for moderate inflations (Lucas (2000)). Clearly, more research is needed to examine the welfare consequences of uncertainty - in particular, one of the lessons of the equity premium puzzle literature is that logarithmic preferences are not consistent with agents' treatment of aggregate risk.

The risk premium associated with the lending contract as well as the price of capital are also monotonically increasing in the variance and the bankruptcy rate. However, note that this is not the case for the last three variables. In particular, the comparison between Economies II and III shows that holding the variance of the technology shock constant but increasing the steady-state bankruptcy rate results in a fall (again relative to Economy II) in both the entrepreneurs' capital stock (*z*) and net worth (*n*). This occurs despite the fact that the price of capital is greater and reflects the impact that the greater bankruptcy rate has on the level of lending in the economy.

¹⁵ In the Carlstrom and Fuerst economy, the minimum technology realization for solvency is: $\bar{\omega} = 0.60$.

We now examine the cyclical behavior of the economy with time-varying uncertainty in the capital production sector.

3.2 Cyclical Behavior

As described in Section 2, eqs. (22) through (30) determine the equilibrium properties of the economy. To analyze the cyclical properties of the economy, we linearize (i.e. take a first-order Taylor series expansion) of these equations around the steady-state values and express all terms as percentage deviations from steady-state values. This numerical approximation method is standard in quantitative macroeconomics. What is not standard in this model is that the second moment of technology shocks hitting the capital production sector will influence equilibrium behavior and, therefore, the equilibrium policy rules. That is, linearizing the equilibrium conditions around the steady-state typically imposes certainty equivalence so that variances do not matter. In this model, however, the variance of the technology shock can be treated as an additional state variable through its role in determining lending activities and, in particular, the nature of the lending contract.¹⁶ Linearizing the system of equilibrium conditions does not eliminate that role in this economy and, hence, we think that this is an attractive feature of the model.

While the previous section analyzed the steady-state behavior of four different economies, in this section we employ the same parameters as in the Carlstrom and Fuerst model (Economy I in the previous section). We depart from Carlstrom and Fuerst by relaxing the *i.i.d.* assumption for the capital sector technology shock. This is reflected in the law of motion for the standard deviation of the technology shock which is given in eq. (30); for convenience this is rewritten below:

$$\sigma_{\omega,t+1} = \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t}^{\zeta} \mu_{t+1}$$

As in Carlstrom and Fuerst, the standard deviation of the technology shock ω_t is, on average,

¹⁶ Specifically, ω_t is assumed to be log normally distributed. Hence, the linear approximation to the equations describing the financial contract (eqs. (24) and (25)) will include the second moment of ω_t .

equal to 0.207. That is, we set $\bar{\sigma}_\omega = 0.207$. We then examine two different economies characterized by the persistence in uncertainty, i.e. the parameter ζ . In the low persistence economy, we set $\zeta = 0.05$ while in the moderate persistence economy we set $\zeta = 0.90$. The behavior of these two economies is analyzed by examining the impulse response functions of several key variables to a 1% innovation in σ_ω . These are presented in Figures 1-3.

We first turn to aggregate output and household consumption and investment. With greater uncertainty, the bankruptcy rate increases in the economy (this is verified in Figure 2), which implies that agency costs increase. The rate of return on investment for the economy therefore falls. Households, in response, reduce investment and increase consumption and leisure. The latter response causes output to fall. Note that the consumption and leisure response is increasing in the degree of persistence. This is not the case, however, for investment - this is due to the increase in the price of capital (see Figure 2) and reflects the behavior of entrepreneurs. This behavior is understood after first examining the lending channel.

The increase in uncertainty affects, predictably, all three key variables in the lending channel: the price of capital, the risk premium associated with loans and the bankruptcy rate. As already mentioned, the bankruptcy rate increases and, in the high persistence economy, this increased rate of bankruptcy lasts for several quarters. This result implies that the bankruptcy rate is countercyclical in this economy; in contrast, in the analysis by Carlstrom and Fuerst the bankruptcy rate was, counterfactually, procyclical.¹⁷ Their focus was on the effects of innovation to the aggregate technology shock and, because of the assumed persistence in this shock, is driven by the change in the first moment of the aggregate production shock. Our analysis demonstrates that second moment effects may play a significant role in these correlations over the business cycle. Further research, both empirical and theoretical, in this area would be fruitful. Returning to the model, the increased bankruptcy rate implies that the price of capital is greater and this increase lasts

¹⁷ In the Carlstrom and Fuerst (1997) model, a technology shock increases output and the demand for capital. The resulting increase in the price of capital implies greater lending activity and, hence, an increase in the bankruptcy rate (and risk premia). Here, greater uncertainty results in greater bankruptcy rates even though investment falls; since labor is also reduced, this produces countercyclical bankruptcy rates and risk premia.

longer in the high persistence economy. The same is true for the risk premium on loans.

Figure 3 reports the consumption and net worth of entrepreneurs in the economies. In contrast to all other variables, persistence has a dramatic qualitative effect on entrepreneurs' behavior. With low persistence, entrepreneurs exploit the high price of capital to increase consumption: the lack persistence provides no incentive to increase investment. Since the price of capital quickly returns to its steady-state values, the increased consumption erodes entrepreneurs' net worth. To restore net worth to its steady-state value, consumption falls temporarily. The behavior in the high persistence economy is quite different: the price of capital is high and forecast to stay high so investment increases dramatically. Initially, the investment is financed by lower consumption, but as entrepreneurs net worth increases (due to greater capital and a higher price of capital) consumption also increases. This endogenous response by entrepreneurs is why, in the high persistence economy, the initial fall in aggregate investment is not as great in the high persistence economy.

A further analysis of the equilibrium characteristics of the high persistence economy is presented in Table 4 in which a few, key second moments are reported. For comparison, the moments implied by the model when subject to total factor productivity shocks (θ_t) or information shocks (σ_ω) are given along with the corresponding moments from the US data. Note that, while time varying uncertainty induces greater volatility in labor, investment, and the capital stock, the discrepancy between the moments from the artificial economy and the actual data are not that much different than arising from a standard RBC model subject to productivity shocks. This behavior stands in stark contrast to the financial intermediation model of Cooper and Ejarque (2000) in which labor and investment were countercyclical and capital stock volatility was over 5 times greater than GDP volatility.¹⁸ Their analysis did not present an explicit model of the financial intermediation sector and our analysis suggests that the endogenous response of this sector to

¹⁸ Cooper and Ejarque (2000) analyze two versions of their model: one in which financial intermediation plays a role in financing both undepreciated and new capital and another in which only new capital (i.e. investment) uses financial intermediaries. The countercyclical behavior of labor and investment is seen in the first version; however, both models exhibit high volatility of the capital stock.

shocks is important and leads to improved performance of the model. The model does imply negative correlation between consumption and investment hence we reach the same conclusion as Cooper and Ejarque (1997): shocks to uncertainty can not be the dominant shock in the economy since this correlation is counterfactual to business cycle behavior. This observation does not, in our opinion, rule out uncertainty as playing a role in business cycle behavior - it simply can not be the sole or dominant factor. A second important feature seen in Table 4 is the quantitatively small role that second moment shocks have on the economy. This is addressed in the next section.

Table 4: Business Cycle Characteristics¹⁹

shocks	σ_y	Volatility relative to y				Correlation with y			
		c	h	i	k	c	h	i	k
θ	0.046	0.63	0.59	2.72	0.87	0.84	0.81	0.91	0.65
σ_ω	0.0007	0.64	1.43	5.14	1.03	-0.54	0.93	0.97	0.36
<i>US data</i> ²⁰	1.71	0.49	0.86	3.15	0.36	0.76	0.86	0.90	-0.08

3.2.1 Quantitative effects

The discussion above demonstrates that changes in uncertainty have potentially important effects on the economy - but it does raise the question, as seen in Table 4, of whether these qualitative effects are quantitatively important. To answer this, we compare impulse response functions of the economy due to an innovation of the aggregate technology shock, θ_t , and uncertainty, $\sigma_{\omega,t}$. Since all variables are measured as percentage deviations from steady-state values, analyzing 1% innovations to both shocks would, at first glance, seem reasonable. Doing so, however, would treat as symmetrical the likelihood of a shock of this magnitude for both innovations. To make the economically meaningful comparison, an estimate of the empirical distribution for the two shocks is required.

¹⁹ For this comparative analysis, the standard deviation of the innovation to both shocks was assumed to be 0.007. This figure is typical for total factor productivity shocks but whether this is a good figure for shocks to the second moments is an open question. We also assumed that both shocks exhibit high persistence with an autocorrelation of 0.95 for θ_t and 0.90 for σ_ω .

²⁰ The US figures are from Kydland and Prescott (1990).

For the aggregate technology shock, we use the Solow residual as measured by:

$$sr_t = y_t - 0.36k_t - 0.64h_t \quad (31)$$

where all variables are measured in per-capita terms and expressed as logs. This series is then linearly detrended and the resulting series identified as θ_t .²¹ While there are many sound reasons why this construction of the aggregate technology shock is flawed, the series can be viewed as a noisy measure of the technology shock.²² Consequently, the volatility of this series is an upper bound of the volatility of the true technology shock.

Measuring the volatility of σ_ω is more problematic. Clearly there is no direct empirical analog to this technology shock to capital production and, while calibration of the parameters of the model employed the average risk premium on commercial paper and the bankruptcy rate, these two series are endogenous and would represent very noisy measures of σ_ω . Instead, we used the data from the Professional Forecaster's Survey (PFS) on the likelihood of various GDP growth rate scenarios. That is, participants in the survey are asked to provide point forecasts for various series such as GDP, unemployment, etc.; in addition, however, they are asked to provide the probabilities associated with the following outcomes for GDP growth (denoted $\% \Delta y$)

$$\begin{aligned} \% \Delta y &> 6, 5.9 > \% \Delta y > 5.0, 4.9 > \% \Delta y > 4.0, 3.9 > \% \Delta y > 3.0, \\ 2.9 &> \% \Delta y > 2.0, 1.9 > \% \Delta y > 1.0, 0.9 > \% \Delta y > 0.0, \\ -1.0 &> \% \Delta y > -0.1, -2.0 > \% \Delta y > -1.1, -2.0 > \% \Delta y \end{aligned}$$

Using the probability weights (and identifying the midpoint of each interval as the outcome, with 6.5% and -2.5% used for the extreme outcomes) the mean and standard deviation for each forecaster can be constructed. We then used the average of these standard deviations as the

²¹ A description of the data is provided in the Appendix.

²² Hoover and Salyer (1998) discuss at length the perils of using the Solow residual to do conditional forecasting with real business cycle models.

measure of σ_ω .²³

Figure 4 presents the empirical density functions for each series. Note that the volatility of σ_ω is much greater than that of θ (for the sample period, $Sd(\sigma_\omega) = 0.072$, $Sd(\theta) = 0.017$). Consequently, the probability mass associated with a $\pm 1\%$ innovation in θ is much larger than that for the σ_ω . To make comparison of the impulse response functions meaningful, we therefore seek the value ϕ such that²⁴

$$\Pr(|\theta| \leq 0.01) = \Pr(|\sigma_\omega| \leq \phi) \quad (32)$$

Using the empirical density functions, the value that satisfies this condition is $\phi = 0.04$; hence, in constructing the impulse response functions, the innovations for the uncertainty shock are 4 times larger than that for the technology shock. The behavior of several key variables (in response to positive innovations in the two shocks) are presented in Figure 5.²⁵

Note that in all cases, the response to a technology shock is greater than that to an increase in uncertainty. This is not surprising in that a technology shock directly affects factor productivity while uncertainty affects the economy through the endogenous response in the lending channel. Yet, the quantitative effects of an increase in uncertainty are not negligible - especially so for the price of capital and the bankruptcy rate.²⁶ Clearly more work is needed to assess the size of the uncertainty shocks that affect the economy. We view these results as suggestive that second moment effects do not have second order effects.

²³ Again, a complete description of the data and the transformations used are in the Appendix.

²⁴ As described in the Appendix, all variables are measured as percentage deviations from sample averages, hence $\sigma_\omega < 0$ is well defined.

²⁵ For these simulations, the persistence of σ_ω was assumed to be 0.90.

²⁶ It is worth noting that we modified preferences by introducing a simple form of habit persistence in consumption. Our thinking was that this may change the behavior of the price of capital (through greater risk aversion) and, therefore, magnify the changes due to greater uncertainty. This was not the case, however; specifically, there was little change in any of the aggregate variables, most notably output and labor. Of course, the welfare consequences of uncertainty would be affected.

4 Conclusion

The effect of uncertainty as characterized by second moment effects has been largely ignored in quantitative macroeconomics due to the numerical approximation methods typically employed during the computational exercise. The analysis presented here uses standard solution methods (i.e. linearizing around the steady-state) but exploits features of the Carlstrom and Fuerst (1997) agency cost model of business cycles so that time varying uncertainty can be analyzed. While development of more general solution methods that capture second moments effects is encouraged, we think that the intuitive nature of this model and its standard solution method make it an attractive environment to study the effects of time-varying uncertainty.

Our primary findings fall into four broad categories. First, we demonstrate that uncertainty affects the level of the steady-state of the economy so that welfare analysis of uncertainty that focus entirely on the variability of output (or consumption) will understate the true costs of uncertainty. Second, we demonstrate that time varying uncertainty results in countercyclical bankruptcy rates - a finding which is consistent with the data and opposite the result in Carlstrom and Fuerst. Third, we show that persistence of uncertainty effects both quantitatively and qualitatively the behavior of the economy. Finally, the magnitude of shocks to uncertainty may be quantitatively large; if so, second moment effects do indeed matter for macroeconomic behavior. Together, these results make a strong case for more research into the effects that uncertainty has on aggregate economic performance. One potentially fruitful area would be to examine the model's implications for asset prices. For instance, it has been noted (see Schwert (1989)) that the volatility of stock returns are countercyclical; it would be interesting to see if this model could produce this behavior.

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5 Appendix:

5.1 Steady-state conditions in the Carlstrom and Fuerst Agency Cost Model

We first present the equilibrium conditions and express these in scaled (by the fraction of entrepreneurs in the economy) terms. Then the equations are analyzed for steady-state implications. As in the text, upper case variables denote aggregate wide while lower case represent household variables. Preferences and technology are:

$$\begin{aligned}U(\tilde{c}, 1 - l) &= \ln \tilde{c} + \nu(1 - l) \\ Y &= \theta K^\alpha [(1 - \eta) l]^{1-\alpha-\phi} \eta^\phi\end{aligned}$$

Where η denotes the fraction of entrepreneurs in the economy and θ is the technology shock. Note that aggregate household labor is $L = (1 - \eta) l$ while entrepreneurs inelastically supply one unit of labor. We assume that the share of entrepreneur's labor is approximately zero so that the production function is simply

$$Y = \theta K^\alpha [(1 - \eta) l]^{1-\alpha}$$

This assumption implies that entrepreneurs receive no wage income (see eq. (9) in C&F).

There are nine equilibrium conditions:

The resource constraint

$$(1 - \eta) \tilde{c}_t + \eta c_t^e + \eta i_t = Y_t = \theta_t K_t^\alpha [(1 - \eta) l_t]^{1-\alpha} \quad (33)$$

Let $c = \frac{(1-\eta)\tilde{c}}{\eta}$, $h = \frac{(1-\eta)l}{\eta}$, and $k_t = \frac{K_t}{\eta}$ then eq(33) can be written as:

$$c_t + c_t^e + i_t = \theta_t k_t^\alpha h_t^{1-\alpha} \quad (34)$$

Household's intratemporal efficiency condition

$$\tilde{c}_t = \frac{(1-\alpha)}{\nu} K_t^\alpha [(1-\eta)l_t]^{-\alpha}$$

Defining $\nu_0 = \frac{\eta}{1-\eta}\nu$, this can be expressed as:

$$\nu_0 c_t = (1-\alpha) k_t^\alpha h_t^{-\alpha} \quad (35)$$

Law of motion of aggregate capital stock

$$K_{t+1} = (1-\delta)K_t + \eta i_t [1 - \Phi(\bar{\omega}_t)\mu]$$

Dividing by η yields the scaled version:

$$k_{t+1} = (1-\delta)k_t + i_t [1 - \Phi(\bar{\omega}_t)\mu] \quad (36)$$

Household's intertemporal efficiency condition

$$q_t \frac{1}{\tilde{c}_t} = \beta E_t \frac{1}{\tilde{c}_{t+1}} \frac{1}{2} q_{t+1} (1-\delta) + \theta_{t+1} \alpha K_{t+1}^{\alpha-1} [(1-\eta)l_{t+1}]^{1-\alpha} \frac{1}{4}$$

Dividing both sides by $\frac{1-\eta}{\eta}$ and scaling the inputs by η yields:

$$q_t \frac{1}{c_t} = \beta E_t \frac{1}{c_{t+1}} q_{t+1} (1 - \delta) + \theta_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \quad (37)$$

The conditions from the financial contract are already in scaled form:

Contract efficiency condition

$$q_t = \frac{1}{1 - \Phi(\bar{\omega}_t) \mu + \phi(\bar{\omega}_t) \mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)}} \quad (38)$$

Contract incentive compatibility constraint

$$\frac{i_t}{n_t} = \frac{1}{1 - q_t g(\bar{\omega}_t)} \quad (39)$$

Where n_t is entrepreneur's net worth.

Determination of net worth

$$\eta n_t = Z_t q_t (1 - \delta) + \theta_t K_t^{\alpha-1} [(1 - \eta) l_t]^{1-\alpha}$$

or, in scaled terms:

$$n_t = z_t q_t (1 - \delta) + \theta_t k_t^{\alpha-1} h_t^{1-\alpha} \quad (40)$$

Note that z_t denotes (scaled) entrepreneur's capital.

Law of motion of entrepreneur's capital

$$Z_{t+1} = \eta n_t \frac{f(\bar{\omega}_t)^{3/4}}{1 - q_t g(\bar{\omega}_t)} - \eta \frac{c_t^e}{q_t}$$

Or, dividing by η

$$z_{t+1} = n_t \frac{f(\bar{\omega}_t)^{3/4}}{1 - q_t g(\bar{\omega}_t)} - \frac{c_t^e}{q_t} \quad (41)$$

Entrepreneur's intertemporal efficiency condition

$$q_t = \gamma \beta E_t \left[q_{t+1} (1 - \delta) + \theta_{t+1} \alpha K_{t+1}^{\alpha-1} [(1 - \eta) l_{t+1}]^{1-\alpha} \frac{q_{t+1} f(\bar{\omega}_{t+1})^{3/4}}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right]$$

Or, in scaled terms:

$$q_t = \gamma \beta E_t \left[q_{t+1} (1 - \delta) + \theta_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \frac{q_{t+1} f(\bar{\omega}_{t+1})^{3/4}}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right] \quad (42)$$

5.2 Definition of Steady-state

Steady-state is defined by time-invariant quantities:

$$c_t = \hat{c}, c_t^e = \hat{c}^e, k_t = \hat{k}, \bar{\omega}_t = \hat{\omega}, h_t = \hat{h}, q_t = \hat{q}, z_t = \hat{z}, n_t = \hat{n}, i_t = \hat{i}$$

So there are nine unknowns. While we have nine equilibrium conditions, the two intertemporal efficiency conditions become identical in steady-state since C&F impose the condition that the internal rate of return to entrepreneur is offset by their additional discount factor:

$$\gamma \frac{\hat{q} f(\hat{\omega})^{3/4}}{1 - \hat{q} g(\hat{\omega})} = 1 \quad (43)$$

This results in an indeterminacy - but there is a block recursiveness of the model due to the calibration exercise. In particular, we demonstrate that the risk premium and bankruptcy rate determine $(\hat{\omega}, \sigma)$ - these in turn determine the steady-state price of capital. From eq.(37)we have:

$$\hat{q} = \frac{\alpha\beta}{1 - \beta(1 - \delta)} \hat{k}^{\alpha-1} \hat{h}^{1-\alpha} = \frac{\alpha\beta}{1 - \beta(1 - \delta)} \frac{\hat{y}}{\hat{k}} \quad (44)$$

From eq.(35)we have:

$$\hat{h} = \frac{1 - \alpha}{\nu_0} \frac{\hat{k}^\alpha \hat{h}^{1-\alpha}}{\hat{c}} = \frac{1 - \alpha}{\nu_0} \frac{\hat{y}}{\hat{c}} \quad (45)$$

From eq.(36)we have:

$$\hat{k} = \frac{1 - \Phi(\hat{\omega})\mu}{\delta} \hat{i} \quad (46)$$

Note that these three equations are normally (i.e. in a typical RBC framework) used to find steady-state $\hat{k}, \hat{h}, \hat{c}$ - because $\hat{q} = 1$. Here since the price of capital is endogenous, we have four unknowns. From eq. (40)and eq. (37)we have

$$\hat{n} = \hat{z} \hat{q} (1 - \delta) + \alpha \frac{\hat{y}}{\hat{k}} = \hat{z} \frac{\hat{q}}{\beta} \quad (47)$$

From eq. (41)and the restriction on the entrepreneur's additional discount factor (eq. (43)), we have

$$\hat{z} = \hat{n} \frac{1}{\hat{q}^\gamma} - \frac{\hat{c}^e}{\hat{q}} \quad (48)$$

Combining eqs. (47)and (48) yields:

$$\frac{\hat{c}^e}{\hat{n}} = \frac{1}{\gamma} - \beta \quad (49)$$

We have the two conditions from the financial contract

$$\hat{q} = \frac{1}{1 - \Phi(\hat{\omega})\mu + \phi(\hat{\omega})\mu\frac{f(\hat{\omega})}{f'(\hat{\omega})}} \quad (50)$$

And

$$\hat{i} = \frac{1}{1 - \hat{q}(1 - \Phi(\hat{\omega})\mu - f(\hat{\omega}))} \hat{n} \quad (51)$$

Finally, we have the resource constraint:

$$\hat{c} + \hat{c}^e + \hat{i} = \hat{k}^\alpha \hat{h}^{1-\alpha} \quad (52)$$

The eight equations (44), (45), (46), (47), (48), (50), (51), (52) are insufficient to find the nine unknowns. However, the risk premium, denoted as ζ , is defined by the following

$$\hat{q}\hat{\omega}\frac{\hat{i}}{\hat{i} - \hat{n}} = \zeta \quad (53)$$

But we also know (from eq.(51) that

$$\frac{\hat{n}}{\hat{i}} = 1 - \hat{q}(1 - \Phi(\hat{\omega})\mu - f(\hat{\omega})) = 1 - \hat{q}g(\hat{\omega})$$

Rearranging eq.(53) yields:

$$\frac{\hat{q}\hat{\omega}}{\zeta} = 1 - \frac{\hat{n}}{\hat{i}}$$

substituting from the previous expression yields

$$\hat{\omega} = \zeta g(\hat{\omega}) \quad (54)$$

Let κ = bankruptcy rate – this observable also provides another condition on the distribution.

That is, we require:

$$\Phi(\hat{\omega}) = \kappa \quad (55)$$

The two equations eq.(54) and eq. (55) can be solved for the two unknowns - $(\hat{\omega}, \sigma)$. By varying the bankruptcy rate and the risk premium, we can determine different levels of uncertainty (σ) and the cutoff point ($\hat{\omega}$).

Note that the price of capital in steady-state, is a function of $(\hat{\omega}, \sigma)$ as determined by eq. (50). The other preference parameter, γ is then determined by eq. (43). Once this is determined, the remaining unknowns: $\hat{c}, \hat{c}^e, \hat{h}, \hat{i}, \hat{k}, \hat{z}, \hat{n}$ are determined by eqs. (44), (45), (46), (47), (49), (51), (52).

Finally, we note that the parameter η does not play a role in the characteristics of equilibrium and, in particular, the behavior of aggregate consumption. This can be seen by first defining aggregate consumption:

$$(1 - \eta) \tilde{c}_t + \eta c_t^e = C_t^A$$

Dividing by η and using the earlier definitions:

$$c_t + c_t^e = c_t^A \quad (56)$$

Since the policy rules for household and entrepreneurial consumption are defined as the percentage deviations from steady-state, aggregate consumption will be similarly defined (and note that since $c_t^A = \frac{1}{\eta} C_t^A$, percentage deviations of aggregate consumption and scaled aggregate consumption are identical). Using an asterisk to denote percentage deviations from steady-state, we have:

$$\frac{\hat{c}}{\hat{c} + \hat{c}^e} c_t^* + \frac{\hat{c}^e}{\hat{c} + \hat{c}^e} c_t^{e*} = c_t^{A*} \quad (57)$$

It is this equation that is used to analyze the cyclical properties of aggregate consumption.

5.2.1 Description of Data

In constructing the Solow residual as defined in eq. (31), the following series were obtained from the US-ECON data set provided by Haver Analytics.

1. GDPH - Gross domestic product (seasonally adjusted at annual rates, billions of chain-weighted 1992 dollars)
2. FNH - Fixed, non-residential investment (seasonally adjusted at annual rates, billions of chain-weighted 1992 dollars)
3. LHTNAGRA - Aggregate hours, wage & salary workers on non-agricultural payrolls (seasonally adjusted at annual rates, millions of hours)
4. LNT20N - Civilian noninstitutional population, both sexes, 20 years and over (thousands, non-seasonally adjusted)
5. EPND - fixed private non-residential capital, billions of dollars.

To construct a quarterly capital stock series for the sample period 1982.1-2002.2, the law of motion for capital was used with fnh identified as investment and the initial capital stock identified as 1981.4 value of $epnd$. The depreciation rate was assumed to be 0.020.

The Professional Forecaster's Survey data was obtained from the Federal Reserve Bank of Philadelphia's website. A complete description of the survey can be found there; in addition, a bibliography of articles that have used this data is also available.

The probabilities attached to various scenarios for real GDP growth over the sample period were used. There were two changes that occurred in the survey in 1992: a finer partitioning of possible outcomes was introduced and the forecast was changed to GDP growth from GNP growth. The latter change is ignored for our purposes while the former makes comparison of the forecasts somewhat problematic. The survey participants were asked to provide probabilities for the

following outcomes (all in percentage terms) in the two periods:

1982 – 1991 :> 6, 4 to 5.9, 2 to 3.9, 0 to 1.9, –2 to –0.1, < –2

1992 – *current* :> 6, 5 to 5.9, 4 to 4.9, 3 to 3.9, 2 to 2.9, 1 to 1.9,

0 to 0.9, –1 to –0.1, –2 to –1.1, < –2

The finer partitioning post 1992 resulted in smaller variances over this period. Hence, to make a consistent series over the entire sample period, standard deviations in each period (the average of the forecasters' standard deviations) were expressed as percentage deviations from the mean in the two samples. This is the series used to construct the empirical analog to $\sigma_{\omega,t}$.

Figure 1: Response of Output, Consumption, and Investment
Low and High Persistence Economies

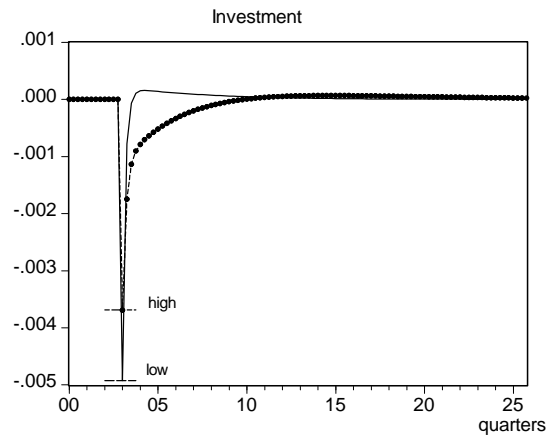
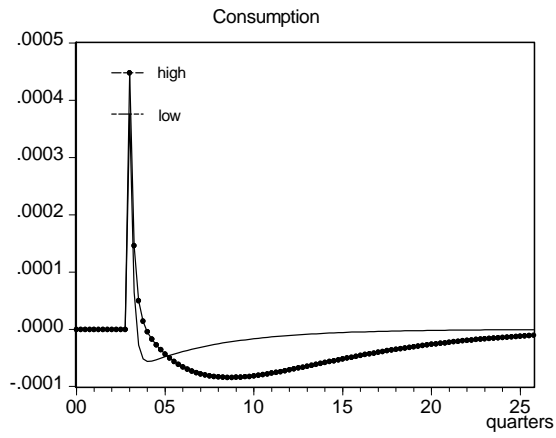
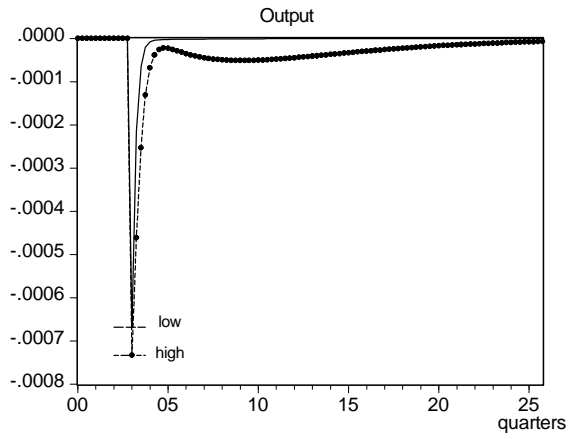


Figure 2: Response of Price of Capital, Risk Premia, and Bankruptcy Rate
Low and High Persistence Economies

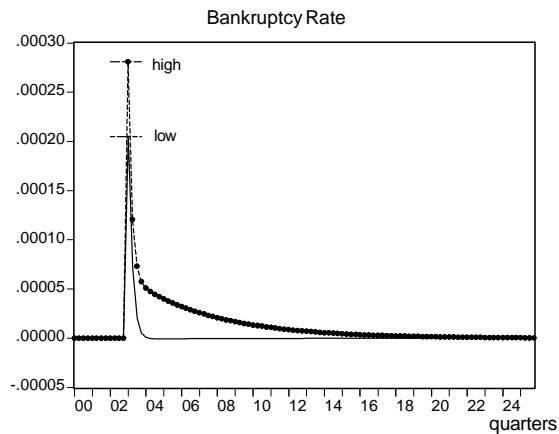
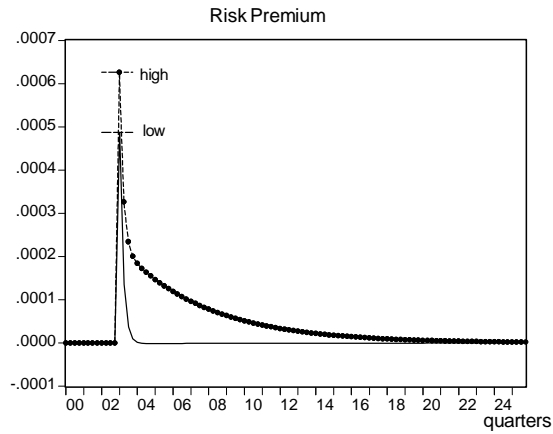
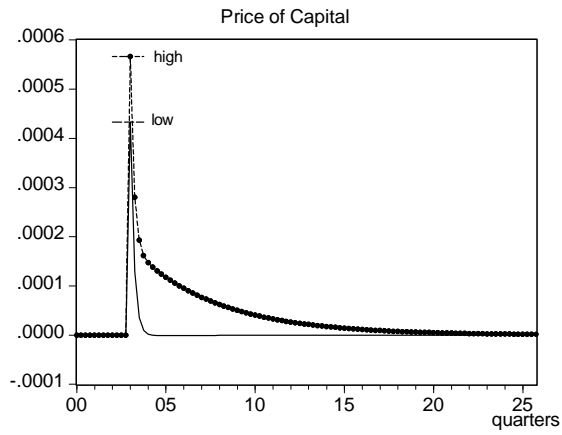


Figure 3: Response of Entrepreneur's Consumption and Net Worth
Low and High Persistence Economies

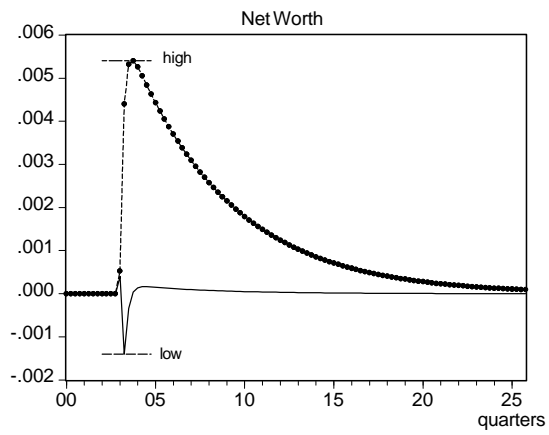
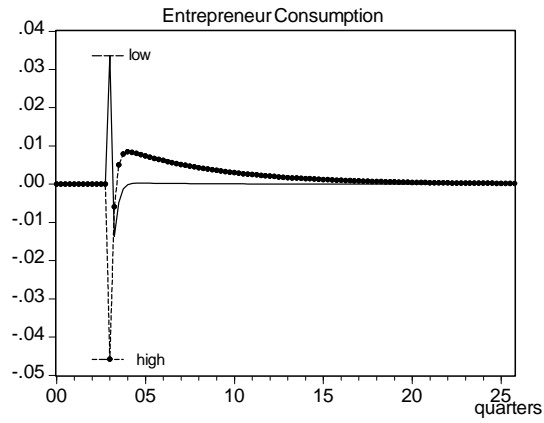


Figure 4: Empirical Density Functions

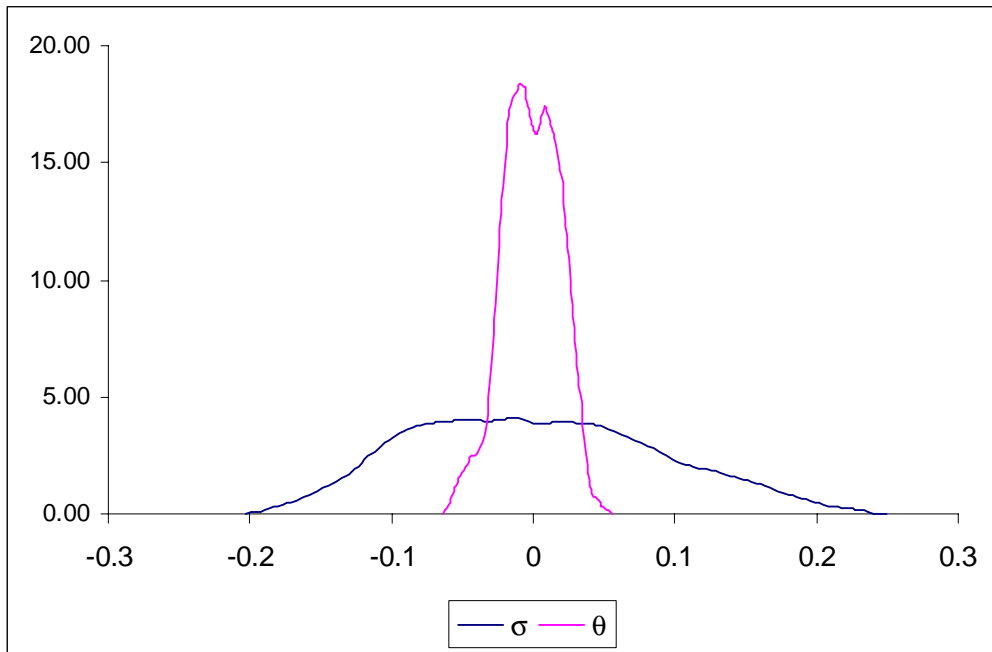


Figure 5: Response to Technology (θ) and Uncertainty (σ_ω) Shock
 (all variables measured as percentage deviations from steady-state)

