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Housing cycles and the period of production

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US fixed residential investment is one of the most periodic economic time series. A theory of housing cycles is analysed on the basis of the period of housing construction. Substantial lags between planning and completion phases of housing construction cause housing investment to respond cyclically to exogenous shocks in demand and production costs. Known structural parameters of the housing industry provide sharp numerical benchmarks for the resulting dynamic system. The calibrated model with two construction lags describes the housing data and cycles well.

I. INTRODUCTION

US residential fixed investment is recognized to be an attractive candidate for studying investment behaviour because of its volatility and data availability (Rosen and Topel, 1989). In this paper, gestation lags (alternatively, time-to-build or construction lags), introduced by the Austrian school in their period of capital production theory and followed by Kydland and Prescott (1982) and Taylor (1982), along with adjustment costs technology are used to understand the cycles in the US quarterly investment in single family housing units. It is found that the long pattern of capital construction lags and expectational (pro-cyclical) investment activity are two main characteristics of the industry that could attribute to its highly volatile investment behaviour. Moreover, a model is presented which produces cycles that fit the data well. More specifically, a housing model with two-period construction lags[1] describes the data. This finding is also reported by Lee (1997), who analyses the empirical form of the US housing supply function.

To show that time-to-build and adjustment cost technologies (or equivalently, a multi-period adjustment cost) help to explain US housing investment cycles, some facts are first presented regarding the US single-family housing industry in the following section. In Section III the model is described and the housing market dynamics analysed. A representative firm's problem is examined, namely that of choosing the initial number of projects to invest in to maximize the present discounted value of its cash flow over an infinite horizon. Investment supply functions are then obtained. Investment depends on housing prices. A single-period adjustment cost technology predicts that only the current (or myopic) price affects investment, while the time-to-build technology predicts that the expectations of future prices – as many lags as there are lags in the technology – should affect current investment. The demand side of the housing market is then discussed. In Section IV, using a two-period lag model, a calibrated model that matches the cycles in the US housing investment is presented. Concluding remarks are presented in Section V. The data are described separately from the main text in a data appendix at the end of the paper.

[1] The actual completion time for single-family housing units is longer if one includes the planning time for land acquisitions, building permits, building itself, and others (i.e. pre-construction lag as well as construction lags *per se*). Such details are ignored mainly due to the lack of data and we focus only on investment in housing structures. Consequently, a housing unit is considered to be started when excavation for the building has begun. Also by convention, one-family unit houses are classified as completed either when all finish flooring has been installed or when the house is occupied even if flooring is not finished. We focus on the construction lags that are naturally associated with the flow of investment in structures. Using a calibrated real business cycle model, Christiano and Todd (1996) show that the pre-construction ('time-to-plan') period may help to account for several key features of business cycles. Their model uses a standard four-period time-to-build investment technology, where the first period is assigned as a time-to-plan stage in which only a negligible amount of resources is used.

II. SOME FACTS ABOUT THE US HOUSING INDUSTRY

One of the prominent features of the US housing industry is that its investment activity is expectational. The Bureau of the Census, Construction Reports C-27, estimates that one quarter of units are built on contract and the rest for the market at large. That is, most of the housing construction firms do not build to order, but instead adjust housing starts and completions in relation to demand conditions and price and cost expectations. Thus, in a time of a housing boom the completion time rate may be accelerated by employing more materials and labour, etc., to ensure quick sales. And in times of recession, firms may lengthen completion lags and may reduce starts. Figure 1 displays the percentage distribution of buildings completed by number of months from start. In recessions (e.g. 1982), firms prolong the construction period by distributing projects in progress over the period, whereas in boom times (e.g. 1977), they finish most of their projects within 6 months. This 'yo-yo' effect of the time-to-build period during recessions and boom periods is also documented by Merkies and Steyn (1994) for the Dutch construction industry. They refer to the contractionary and expansionary effect over the business cycles as the 'accordion effect'.

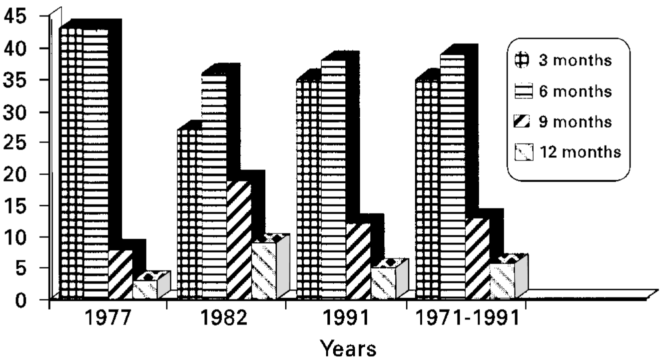

Fig. 1. *Percentage distribution of buildings completed by number of months from start for residential buildings with one unit. Source: US Department of Commerce, the Bureau of the Census. Construction Reports C20, Supplement 2. Total Time From Start of Construction to Completion of Private Residential Buildings. (Note: the percentage distribution is not adjusted upward to reach the 100 per cent value.)*

The other feature of the industry is its long pattern of capital construction lags. Housing construction is characterized by long gestation periods and a long response time for stocks to adjust to new market conditions. There are other US industries, for example US manufacturing plant or plant additions (Mayer, 1960; Altug, 1989), a power generating plant (MacRea, 1989), and non-residential construction (Montgomery, 1995) that exhibit the time-to-build pattern. The single-family unit housing construction industry shows a clear pattern of time-to-build and the data are among the best available to study it. For example, both market output and input prices of capital are directly observed in the housing sector. The Bureau of Census reports the total time from start of construction to completion of private residential buildings in a special supplement to the publication of Housing Starts (Construction Reports, C20). Table 1 summarizes the average completion time for residential structures from 1971 to 1992. It shows that completion time is approximately 6.12 months.

This average completion time over three decade 'hides a great deal of variation cross-sectionally as well as from year to year' (Bils and Kahn, 1994, p. 21). The data on the percentage distribution of buildings completed by number of months (see either Table 2 or Fig. 1) show that on average three-quarters of buildings are completed within 6 months,

Table 1. *Average number of months from start to completion of new one-family houses completed, by region and purpose of construction*

Year[a]	Region[b] United States	N.E.	South	Average unit size (sq. ft.)	Aggregate (US) investment[c] ($bil)
1971–1979	5.82	6.3	5.45	1600	44.9
1980–1989	6.37	7.04	5.68	1800	84.1
1990–1992	6.17	8.6	5.43	2100	102.1
1971–1992	6.12	7.31	5.52	1850	77.03

Source US Department of Commerce, the Bureau of the Census. Construction Reports C20, Supplement 2, *Total Time From Start of Construction to Completion of Private Residential Buildings*. And C25, Supplement A, *Characteristics of New Housing*.
[a]Although the actual estimation is done over the time period of 1963:1 to 1991:4, years 1963–1969 could not be reported in this table since the first survey for Supplement 2 for Construction Reports C20 starts from 1971.
[b]The original survey in Table S2-1 reports other regions as well, namely, Midwest and West. Moreover, the average number of months in the table is for all purpose of construction, which includes the houses built for sale, contractor built houses, and owner built houses.
[c]Aggregate US investment refers to the actual investment in structures for a single family unit housing.

Table 2. *Percentage distribution of buildings completed by number of months from start for esidential buildings with one unit*

	Period			
Year	3 months	6 months	9 months	12 months
1971	43	43	8	3
Cumulative	43	86	90	97
1980	27	36	19	9
Cumulative	27	63	82	91
1991	35	38	12	5
Cumulative	35	73	85	90
1971–1991	35	39	13	5.7
Cumulative	35(37.8)[a]	74(80)	87(93.9)	92.7(100)

Source US Department of Commerce, the Bureau of the Census. Construction Reports C20, Supplement 2, *Total Time From Start of Construction to Completion of Private Residential Buildings*.
[a]The numbers in brackets are adjusted upwards to reach the 100% value.

although in recessions (e.g. 1982) buildings take longer to complete. A longer completion time in recessions suggests that some firms may face liquidity constraints, but a more convincing reason for the variation of completion time from year to year is given by Bils and Kahn. They find a strong negative correlation, − 0.8, between time-to-build and the increase in the number of starts over the preceding year. This suggests that people 'expect time-to-build to respond more to unexpected movements in sales and production, which are presumably better approximated by the growth in starts' (Bils and Kahn, 1994, p. 21).

Besides the economic reasons, the stability in completion time is also partially due to offsetting effects between improved construction technology (shortening effect) and changing characteristics of single-family unit housing (extending effect). The 'characteristics of new housing,'[2] which are used to calculate the hedonic housing prices, have changed from 1970 to 1990. For example, over the last three decades, the average unit size of one family unit increased from roughly 1600 to 2100 square feet. Furthermore, housing details, such as heating system and central air-conditioning, have become much more sophisticated and much harder to install.

In summary, it is clear that the time-to-build aspect of new houses is a considerable and significant part of the US housing industry. Moreover, the amount of investment and the longer completion time of construction are positively correlated with boom and recession periods, respectively. With these facts in hand, it seems thus logical to explicitly incorporate both time-to-build and adjustment cost to capture the lengthening and shortening effects as well as the construction lags when one analyses housing investment models.

III. INVESTMENTS: SUPPLY OF NEW HOMES

Housing investment is the main series to be explained. Figure 2 graphs annual time series of investment, input and output prices for houses and shows that every downturn in housing investment is associated with the National Bureau of Economic Research's definition of a peak-to-trough period of the business cycle. NBER's peak-to-trough periods are highlighted over the time series in Fig. 2. The first small downturn occurs in the 1968–70 recession. A larger down ward course happens during the 1973–75 period, with a few small movements before. The major boom and bust in construction activity occurs during the 1977–83 cycle, with the boom in housing investment reached in 1977–78. It takes a drastic downward course in 1980 and bottoms out in November 1982. The last major peak in construction occurs in 1987.

Figure 2 also shows that housing investment activity leads the peak-to-trough cycle by three to six months. That is, a downturn in housing investment is well in process before the peak in the business cycle occurs, and there is an upswing in housing investment during the trough period. The housing industry seems to invest with two-quarter leads over the general economic conditions. This point is discussed in more detail in the following section.

It is elementary economics that high house prices relative to input costs increase construction activities. As we are concerned with models that take the supply curve to be fixed, the rising supply price of housing investment is the underpinning factor of the theoretical model developed in this paper. Annual time series of investment, input, and house prices are used in Figs. 2 and 3, to present clear

[2]This is defined in the Current Construction Report C.25, *New One-Family Houses Sold and For Sale* – special supplement.

evidence of an upward sloping investment supply function, although the quarterly data from 63 : 1 to 91 : 4 are used to calibrate the detailed model. The quarterly series are too volatile seasonally to depict the salient longer term positive correlations that exist between price and investment. The time series in Fig. 2 provides informal evidence that 'cyclical movements in housing construction are driven largely by

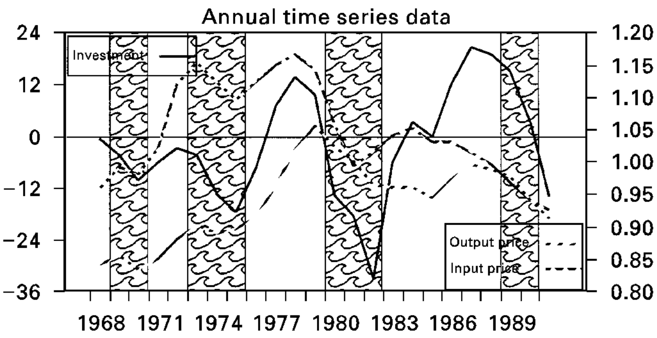

Fig. 2. *Housing investment versus input and output prices. Note: Shaded areas are the periods of peak to trough which is defined by National Bureau of Economic Research.*

'Investment' is the fixed residential investment in single family structures. Sources: US Bureau of Economic Analysis, The National Income and Product Accounts of the United States. (Investment is detrended). base year = 87

'Input price' is the Boeckh Cost Index, which is a weighted average of construction input prices for small residential structures. Source: US Department of Commerce, Bureau of Industrial Economics, Construction Review. Deflated by C.P.I. base year = 87

'Output price' is the New single family housing prices. Source: US Bureau of the Census, New One-Family Houses Sold and for Sale (Construction Reports, ser. C25). Deflated by C.P.I. base year = 87

demand fluctuations along a rising supply curve of new homes' (Rosen and Topel, p. 718). Figure 2 shows that the correlation between annually detrended investment in housing structures and real price in housing is 0.79. This real price in housing is a hedonically adjusted house with 1987 characteristics deflated by the consumer price index. The co-movements of a Boeckh index of construction costs, investments, and housing prices lend additional support for the hypothesis of a rising supply price (e.g. correlation values are 0.77 and 0.75 between investment and construction costs, and housing prices and construction costs, respectively). The co-movements of three series, however, between 1981 and 1982 are not supported. No explicit economic reasons have been found for this. Nevertheless, a more convincing upward sloping supply function that is not affected by a time trend is presented in Fig. 3, where the home prices and investments are plotted against each other.

Multi-period adjustment costs model

Although the supply function is the main focus of this paper, it is only a part of a structural model of the overall market. Thus, to understand the dynamics of the housing market, a more general model is initially outlined.

The best way to analyse market dynamics is to apply a simple but general enough model to approximate the actual function in a fairly wide range of empirical situations. In doing so, tax (Chrinko, 1987), credit availability (Poterba, 1984; Stein, 1993), inflation (Kearl, 1979) and demographic issues in housing investment (Poterba, 1991) are ignored.

The market equilibrium analysis begins with the supply side of the model. Consider a representative firm which is rationally managed in a competitive industry. Thus the

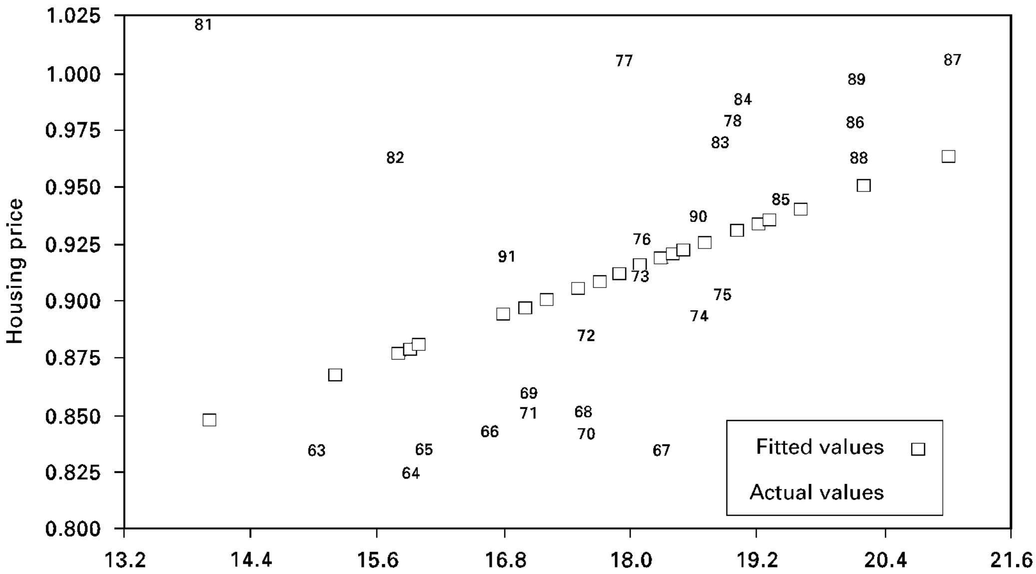

Fig. 3. *Housing investment supply (annual data 1963–1991). Investment used in this graph is in level. Source: See Fig. 2. Housing Price is 1987 hedonic prices.*

output price P_t is exogenous. Given that firm level time-series data do not exist, the representative firm's problem is considered using industry-level data. The US private residential construction industry can be described as being unconcentrated, with a large number of small construction firms which remained stable from 1971 to 1990 (hence supporting the 'price taker' assumption). Further, the difference in the completion time (stated in Table 2) between Northeast (7.31) months) and South (5.53 months) in the period 1971 to 1991 is mainly due to climate difference. Quarterly investment data shows clear seasonal variations: summertime construction is twice as large as in winter (this is not plotted). Thus, the regional difference on the completion time cannot be attributed to heterogeneous firms.

As previous research has rejected the single period adjustment cost model, the time-to-build is blended with adjustment cost to avoid the problems faced by adjustment cost models. However, in order to avoid the 'fixed plan' problem associated with time-to-build technology, the firm is allowed to make changes in the original plan but with costs. In doing so, the model becomes an extension of multi-period adjustment costs theory. The model, however, is not as 'pure' as the multi-period model proposed by Park (1984), who allows changes in the original plan to occur in the latter stages of production. The adjustment process in this paper deals with changes in new investments that can only be initiated at the beginning of production. That is, once the production has started, the firm cannot hurry-up a production in the original plan even if it has enough time to do so. Rather, it can only expand the whole production set by starting a new set of investments. Consequently, the 'accordion effect' of construction activities in the industry is not addressed in the model.

The supply side of this multi-period model begins with a construction of capital technology, in which a single-family unit house takes two periods to build. But any fixed period of production will do. The data on the percentage distribution of buildings completed in Table 2 shows that it is reasonable to consider a case for two-period (quarter) projects as the average completion time is 6.12 months (two quarters). Gestation lags longer than two periods are mere extensions of a two-period model. Thus, the inclusion of longer lags does not change the essence of the issue but only adds computational complexity.

Accordingly, it is supposed that firms invest in projects that take two periods to complete. Following the time-to-build literature, let S_t be the number of projects started at time t. That is, projects that are two stages from completion. S_{t-1} is the output plan of the last period; that is in the final stage of production. Thus the following stock-flow equation governs the capital (house) accumulation for the housing construction industry:

$$K_{t+2} = (1-\delta)K_{t+1} + S_t \tag{1}$$

where K_t is the number of complete houses in the entire housing stock at time t, and housing depreciates at rate $\delta \in (0, 1)$. The depreciation rate can be thought of as the maintenance rate of houses at t. Equation 1 says that projects started today, S_t, are added to the stock of productive capital two periods later.

The firm also faces a current investment constraint. The current gross investment consists of the value put in place during different stages of production.

$$I_t = \omega_1 S_t + \omega_2 S_{t-1} \tag{2}$$

where ω_j denotes the fraction of resources allocated to the investment projects at the jth stage from completion. In this work, ω_1 and ω_2 are taken as exogenously determined fixed parameters. Thus, the definition of ω_j, implies that

$$\omega_1 + \omega_2 = 1 \tag{3}$$

Substituting Equation 1 into Equation 2, the gross investment expenditure in period t affects the availability of capital goods in the future. That is,

> Total gross investment = [value put in place during the first quarter of projects started in the current period] + [value put in place during the second quarter of projects started in the previous period].

The representative firm in a competitive market generates revenues by selling the number of houses completed during time t. Consequently, the revenue function is given by $P_t S_{t-2}$, where P_t is the price of a house (i.e. real hedonic price index for 1987 quality homes) and S_{t-2} is the number of houses started two periods ago and ready to be sold at time t (housing investment is measured in billions of 1987 dollars).

Operating costs of construction arise from purchasing inputs I_t and the vector of variables Y_t that shift the cost function. Y_t can be thought of as the cost shock. The firm's cost function follows a quadratic form so that the marginal cost of installation is a linear increasing function of the level of gross investment: For $I > 0$, $C(I, Y) > 0$, $C_1(I, Y) = \partial C/\partial I > 0$, and $C_{11}(I, Y) = \partial^2 C/\partial I^2 > 0$ with $C(0) = 0$. The simplest function with these characteristics is the quadratic

$$C(I_t, Y_t) = aI_t + a_0 I_t Y_t + \frac{b}{2} I_t^2 \tag{4}$$

where b represents the adjustment cost parameter that makes marginal cost an increasing function of I_t. The costs of adjusting the capital stock increase at an increasing rate with the absolute value of the rate of expansion. The adjustment costs are made a function of cost investment I_t instead of net capital ΔK because the rising supply price refers to the resources needed to build new houses plus depreciation. With the adjustment cost specification above, a speedy adjustment of capital stock to the 'desired' level is more costly than a slow one so long as $b > 0$. The adjustment costs employed here reflect the internal costs to the construction industry but external to firms. These are the costs which are associated with the change of capital stocks when there are technology and other supply shocks. Had the model been an aggregate one (i.e. for the economy as a whole) then the adjustment costs might be referred to as external costs (Mussa, 1977).

Derivation of the supply function

In making its supply decision, the firm maximizes the discounted present value of all future net cash flow. The firm then chooses the number of first-stage investment projects S_t in the maximization problem as expressed below

$$\underset{\{S_{t+\tau}\}}{\text{Max}}\, E_t \sum_{t=0}^{\infty} \left(\prod_{i=0}^{\tau} \beta_{t+i} \right) [P_{t+\tau} S_{t+\tau-2} - C(I_{t+\tau}, Y_{t+\tau})] \quad (5)$$

subject to Equations 1, 2 and 3. $\beta = 1/(1+r)$ is the discount factor and r is the average real interest rate. E_t denotes expectations conditional on the date t information set, which includes the past and present values of housing prices. Holding the discount factor constant[3] and differentiating with respect to S_t, the supply decision rule then is

$$\beta^2 E_t(P_{t+2}) = E_t[\omega_1 C_1(I_t, Y_t) + \beta\omega_2 C_1(I_{t+1}, Y_{t+1})] \quad (6)$$

The left-hand side of Equation 6 is the discounted expected marginal revenue that is to be realized at the end of two periods. The right-hand side is the sum of the discounted expected marginal costs until the period before the completion date.

For gestation lag of length J (i.e. investment projects take J periods to complete), Equation 6 simply becomes

$$\beta^J E_t(P_{t+J}) = E_t\left[\sum_{j=0}^{J-1} \beta^j \omega_{j+1} C_1(I_{t+j}, Y_{t+j}) \right] \quad (7)$$

That is, the discounted expected marginal revenue at J period equals the sum of the expected discounted marginal costs until period $J-1$.

On the other hand, if $J = 0$ then Equation 7 simplifies to the usual adjustment cost decision rule derived from the condition that current price equals current marginal cost:[4]

$$P_t = C_1(I_t, Y_t) \quad (8)$$

The supply function for the two-period model is obtained by inverting Equation 6 so that

$$I_t = S(E_t[P_{t+2}, I_{t+1}, Y_t, Y_{t+1}]) \text{ with } \frac{\partial S}{\partial E_t(P_{t+2})} > 0 \quad (9)$$

Thus, the production side of the model can be simply described by combining Equations 1 and 9:

$$K_{t+2} = (1-\delta)K_{t+1} + S(E_t[P_{t+2}, I_{t+1}, Y_t, Y_{t+1}]) \quad (10)$$

where the total number of houses in stock today equals the depreciated number of housing stocks from the period before plus the houses that were started two periods ago.

By employing gestation lags technology, the solution to the optimization problem implies that current total gross investment depends on the current expectation that is projected on J future price, plus the sum of expected future investment and cost shifters until period $J-1$. As a result, one of the features of general form of Equation 9 is that when the time-to-build technology is used, no more than the pre-specified J lags appear between the expectation operator and the housing price, and $J-1$ lags with respect to investment and cost shifters in the gross investment schedule. Hence, one can see that current price P_t no longer incorporates all current and future information that is relevant for the investment decision, I_t: expectations of future prices and other variables affect current supply. Consequently, my model sheds new light in addressing the positive serial correlation puzzle in the investment literature.[5]

Moreover, this model is a formalization of the fact that the US single-family housing investment cycles precede business cycles by at least three months. Although the economy might be in a recession and there are many unsold houses, one could observe a boom in the single-family construction industry due to an anticipation of an increase in future demand. Situations of excess supply in real estate is

[3] Although the value of expected cash flows is discounted by a time-varying rate in the objective function, it is assumed that the firm is owned by risk-neutral individuals. Consequently, the firms discount factor is assumed to be constant. Using a time-varying discount factor may give the firm an added instrument to hedge against either highly pro-cyclical or counter-cyclical cash flows but this is too difficult to work with empirically.

[4] For clarification, the term 'single-period adjustment cost model' is used for $J = 0$. That is, $J = 0$ is meant to imply that investment projects are finished within a period rather than to unrealistically indicated that they take no time to complete. Consequently, if $J = 1$ so that the output is not realized until the next period (i.e. one-period model), then the supply decision becomes $\beta E_t(P_{t+1}) = C_1(I_t, Y_t)$. Thus, the current investment depends on the discounted one-period-ahead expected price.

[5] The second motivation for this paper is due to the empirical failure of the Q-theory of investment (Abel, 1980; Summers, 1981; Hayashi, 1982; Abel and Blanchard, 1986). The house prices in this paper play a similar role to that of the corporate stock prices in Tobins Q investment model. The Q hypothesis says that investment should be an increasing function of Q. With a convex adjustment costs in current rates of change only, the Q-theory implies that only the current Q should affect the optimal rate of investment (Hayashi, 1982). However, although the current Q is a significant indicator for investments, empirically when investment is regressed on current Q, one often sees low explanatory powers and serial correlation is residuals.

not uncommon. Kling and McCue (1987) note the over-building phenomenon for new office building construction. They observe that high investment continued in new office construction (around \$30 billion in the first quarter of 1985) in spite of the high vacancy rate (around 16–18%). This over-building 'puzzle' during recession is possible when the firm's decision to invest depends, due to construction lags, on the future expected market price as well as other future variables. An optimizing firm would invest to build in spite of unfavourable current economic conditions in order not in miss the forthcoming improved market condition and benefit from it. Of course, the time-to-build aspect is only one of many possible explanations for this over-building 'puzzle'. Poor planning on the firm's part, tax issues concerning construction, and other financial aspects of the firm could contribute to this puzzle as well.

Housing demand

To analyse the demand side of the market, the problem of matching buyers and sellers to specific houses is ignored. In particular, a perfect capital market, a costless auction market and houses that are homogeneous are assumed, so matching considerations are not needed.

The inverse demand function for housing services is a decreasing function of the stock of housing K_t, and can be written as

$$R_t = F(K_t, X_t) \quad \text{with} \quad \frac{\partial F}{\partial K} < 0 \qquad (11)$$

where R_t is the flow (rental) price of housing and X_t is a vector of exogenous demand shifters such as changes in consumer income.

The demand side is completed by the relationship between the rental price, R_t, and the asset or stock price of a house. The standard one-period formula for the asset value of a house in the present value of the expected future rental income of owing it, or

$$P_t = E_t(R_t + \rho R_{t+1} + \cdots) \quad \text{where} \quad \rho = \frac{1-\delta}{1+r} \qquad (12)$$

Using the forward operator, where $L^{-k}R_t = R_{t+k}$, Equation 12 can be equivalently expressed as

$$R_t = P_t - \rho E_t(P_{t+1}) \qquad (13)$$

Considering only the interest rate and depreciation, the rental cost of a house equals the current house price less the discounted nominal price appreciation of the asset. Equating Equation 11 with 13 completes the demand side of the model

$$F(K_t, X_t) = P_t - \rho E_t(P_{t+1}) \qquad (14)$$

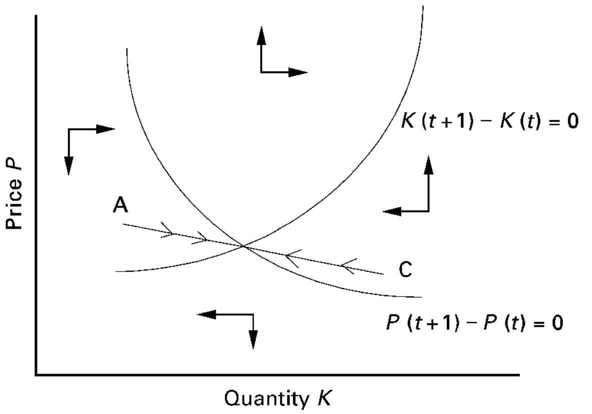

Fig. 4. *Phase diagram for the housing model*

Market equilibrium

The complete market dynamics of stocks and prices are described by Equations 10 and 14 along with the transversality condition that the value of housing is bounded:

$$K_{t+2} = (1-\delta)K_{t+1} + S(E_t[P_{t+2}, I_{t+1}, Y_t, Y_{t+1}]) \quad \text{(supply side)}$$

$$F(K_t, X_t) = P_t - \rho E_t(P_{t+1}) \quad \text{(demand side)}$$

$$\lim_{\tau\to\infty} \rho^\tau P_t K_t = 0 \qquad (15)$$

To understand the dynamics of the housing market, the second-order two-equation system (Equations 10 and 14) is analysed as in Sheffrin (1983), using the phase diagram shown in Fig. 4. The supply Equation 10 can be re-arranged so that in equilibrium the housing stock remains constant:

$$K_{t+2} - K_{t+1} = 0 \Leftrightarrow \bar{K} = \frac{S(\bar{P}, \bar{Y})}{\delta}$$

where, the variables with the bar above denote a steady state. The $K_{t+2} - K_{t+1} = 0$ locus is clearly upward sloping since $\partial S(\cdot)/\partial P$ is positive. But this upward sloping is nothing more than re-stating the equilibrium condition where at 'higher stocks of housing, depreciation is higher, and therefore a higher housing price is needed to bring forth enough production to equal the higher depreciation' (Sheffrin, p. 174). If the price is above the locus then production has to increase and consequently the housing stock increases.

The steady state demand curve for the housing market, $P_{t+1} - P_t = 0$, in equilibrium can also be re-arranged as

$$P_{t+1} - P_t = 0 \Leftrightarrow \bar{P} = \frac{F(\bar{K}, \bar{X})}{1-\rho} \qquad (16)$$

Since P_t does not change at the steady state there are no anticipated capital gains. The locus, $P_{t+1} - P_t = 0$, is obviously downward sloped since $\partial F/\partial K$ is negative. Above this locus, there is an upward pressure since the rental cost increases in conjunction with housing price increase.

As in the discussion by Sheffrin, the housing market with perfect foresight assumption exhibits saddle point instability with the stable arm A–C being downward sloping. If there is an equilibrium disturbance, the stable arm of A–C is the only path that leads back to the steady state in the long run. Clearly, the dynamics and cycles of the market can be observed. For example, if there is an anticipated transitory increase in the future demand, then house prices increase immediately and this consequently leads to increased construction activities prior to when the change actually occurs. With the accumulation of stock, the rental prices fall. When the actual change in demand occurs, rational market participants know that housing prices will fall towards equilibrium as new construction activities increase the stock. On the supply side, after the shock has passed and demand returns to its original levels, the housing stock is too large and thus has to decrease to the steady state values. Price drops thereafter to discourage construction about the prices and investment gradually increase back to the steady state levels.

IV. CALIBRATING THE TWO PERIOD MODEL

To understand the cyclical behaviour of the US housing industry, the two-period gestation lag model discussed in the previous section is calibrated and it is shown that the cycles in housing investment occur because of gestation lags. Even without calibration, one can foresee the cyclical behaviour of the industry when there are gestation lags in construction: any ex-post mis-allocation of resources in the first stage of projects will lead to the contraction or expansion of investment in the subsequent periods to finish the projects and to correct past mistakes. The dis-allocations, consequently, drive down the supply of new houses.

Collecting results from above, the complete market equilibrium with two-period gestation lags can be described by the following equations:

$$\beta^2 E_t(P_{t+2}) = E_t[\omega_1 C_1(I_t, Y_t) + \beta\omega_2 C_1(I_{t+1}, Y_{t+1})]$$

$$I_t = \omega_1 S_t + \omega_2 S_{t-1}$$

$$K_{t+2} = (1-\delta)K_{t+1} + S(E_t[P_{t+2}, I_{t+1}, Y_t, Y_{t+1}]) \quad \text{(supply side)}$$

$$F(K_t, X_t) = P_t - \rho E_t(P_{t+1}) \quad \text{(demand side)}$$

The supply side is represented by the first three equations, with the fourth equation capturing the demand side of the market. For exposition purposes, both the supply and demand shifters as well as the expectation operator are ignored when calibrating the model. To eliminate I_t, S_t and S_{t-1}, it is found that P_t and K_t evolve according to

$$\beta^2 P_{t+2} = \omega_1 \frac{\partial C}{\partial I}(\omega_2 L + \omega_1)(K_{t+2} - (1-\delta)K_{t+1})] + \beta\omega_2 \frac{\partial C}{\partial I}[(\omega_2 L + \omega_1)(K_{t+3} - (1-\delta)K_{t+2})] \tag{17}$$

where L is the lag operator. Taking linear approximations, assuming the cost function is quadratic

$$C(I_t) = aI_t + \frac{b}{2}I_t^2 \tag{18}$$

and the inverse demand function for housing services $F(K)$ is linear

$$F(K_t) = A_0 - A_1 K_t \tag{19}$$

Equations 17 and 14 become (ignore a and A_0 since they affect only levels)

$$\beta^2 P_{t+2} = \omega_1 b[(\omega_2 L + \omega_1)(L^{-2} - (1-\delta)L^{-1})K_t] + \beta\omega_2 b[\omega_2 L + \omega_1)(L^{-3} - (1-\delta)L^{-2})K_t] \tag{20}$$

$$P_t = -(1-\rho L^{-1})^{-1} A_1 K_t \tag{21}$$

Substituting for P_t, K_t evolves according to an autonomous fourth-order linear difference equation

$$\begin{aligned} -\beta^2 L^{-2}(1-\rho L^{-1})^{-1}\frac{A_1}{b}K_t &= \omega_1[(\omega_2 L + \omega_1)(L^{-2} - (1-\delta)L^{-1})K_t] \\ &\quad + \beta\omega_2[(\omega_2 L + \omega_1)(L^{-3} - (1-\delta)L^{-2})K_t] \end{aligned} \tag{22}$$

$$\Leftrightarrow$$

$$\begin{aligned} 0 &= \beta^2(1-\rho L^{-1})^{-1}\frac{A_1}{b}K_t \\ &\quad + \omega_1 L[(\omega_2 L + \omega_1)(L^{-1} - (1-\delta))K_t] \\ &\quad + \beta\omega_2[(\omega_2 L + \omega_1)(L^{-1} - (1-\delta))K_t] \end{aligned} \tag{23}$$

Equation 23 can be simplified by grouping the term

$$\begin{aligned} &(\omega_2 L + \omega_1)(L^{-1} - (1-\delta)) \\ &= [\omega_2 - \omega_1(1-\delta)] + \omega_1 L^{-1} + \omega_2(1-\delta)L \\ &\equiv \alpha_1 L^{-1} + \alpha_2 + \alpha_3 L \end{aligned}$$

where, $\alpha_1 = \omega_1$, $\alpha_2 = [\omega_2 - \omega_1(1-\delta)]$, $\alpha_3 = \omega_2(1-\delta)$. This simplification then leads

$$0 = \beta^2\frac{A_1}{b}K_t + (\omega_1 L + \beta\omega_2)(1-\rho L^{-1}) \times (\alpha_1 L^{-1} + \alpha_2 + \alpha_3 L)K_t \tag{24}$$

Further simplification can be achieved by noting that

$$\begin{aligned} &(\omega_1 L + \beta\omega_2)(1-\rho L^{-1}) \\ &= -\beta\omega_2\rho L^{-1} + \beta\omega_2 - \rho\omega_1 + \omega_1 L \\ &\equiv \gamma_1 L^{-1} + \gamma_2 + \gamma_3 L \end{aligned}$$

Table 3. *Summary statistics for the quarterly data used in the estimation*: *1963:1 to 1991:1 with 116 quarters*

Variables	Mean	Std
Investment	54.82	32.2
House price (*HP*)	0.921	0.07
Real interest rate (*RIR*)	3.402	2.02
Inflation rate (*IR*)	1.013	0.008
Months (*M*)	3.586	0.69
Consumption (*C*)	41.1	10.5
Family (*F*)	7.51 + e04	1.27 + e04
Energy (*E*)	0.82	0.127
Mortgage rate (*MR*)	9.06	2.32

Correlations

	HP	*RIR*	*IR*	*M*	*C*	*F*	*E*	*MR*
HP	1	0.61	0.48	0.39	− 0.79	0.79	0.65	0.74
RIR		1	0.005	0.23	− 0.85	0.89	0.44	0.61
IR			1	0.60	− 0.09	0.15	0.31	0.38
M				1	− 0.29	0.28	0.31	0.35
C					1	− 0.95	− 0.71	− 0.79
F						1	0.52	0.70
E							1	0.87
MR								1

For definitions of these variables, see the Data Appendix.

where, $\gamma_1 = -\beta\omega_2\rho$, $\gamma_2 = \beta\omega_2 - \rho\omega_1$, $\gamma_3 = \omega_1$. Thus, Equation 24 expressed as

$$0 = \left[\beta^2 \frac{A_1}{b} + (\gamma_1 L^{-1} + \gamma_2 + \gamma_3 L)(\alpha_1 L^{-1} + \alpha_2 + \alpha_3 L)\right] K_t \tag{25}$$

Finally, expanding and re-arranging the second term in Equation 25, the characteristic equation of this linear fourth-order difference equation is:

$$\alpha_1\gamma_1\lambda^2 + (\alpha_2\gamma_1 + \alpha_1\gamma_2)\lambda + \left(\beta^2\frac{A_1}{b} + \alpha_3\gamma_1 + \alpha_2\gamma_1 + \alpha_1\gamma_3\right) + (\alpha_2\gamma_1 + \alpha_3\gamma_2)\lambda^{-1} + \alpha_3\gamma_3\lambda^{-2} = 0 \tag{26}$$

where λ is a characteristic root of this system.

To obtain the numerical characteristic roots of this fourth-order equation values need to be assigned to all the structural parameters as follows:

- The value 1/2 is assigned to both ω_1 and ω_2. This value is taken from Table 3. The percentage distribution of buildings completed by the end of three months (35%) is approximately the same as that for six months (39%).
- The quarterly depreciation rate, δ, is calculated at 0.0035 per quarter using a perpetual inventory method.
- The real interest rate (3 month T-Bills deflated by CPI) r is set to 3.4%.
- The adjustment cost parameter b is obtained by using the supply function (Equation 8) and the cost function in Equation 4. With the values of price[6] P set to average 100 thousand dollars and investment I to 550 thousand starts,[7] the parameter value b (dollars per start) then equals 0.018 (i.e. $b = \bar{P}/\bar{I}$).
- The slope of the rental demand function, A_1, is calculated as follows. Assuming that the rental demand has a unit elasticity[8] and using Equation 19, A_1 (dollar per house) then equals to $\bar{R}/\bar{K}$
- The rental price R is approximated using Equation 13 at steady state: $R = P(1 - \rho)$. With $1 - \rho = (r - \delta)/(1 + r) \approx 0.0295$. Thus R is approximately 2950 dollars per quarter.
- To calculate the long-run or steady-state capital stock K it is set equal to I/δ. Since the number of housing starts is 550 thousand per year, the number of quarterly starts is 1.375×10^5. Thus the steady-state capital stock is approximately 39.3 million houses.

[6] This is an average sales price for the kind of houses sold in 1987 (estimated from the hedonic price index). The actual average price over the period is 97 448 dollars for a single-family unit house.
[7] The number of housing starts is calculated from the gross investment in structures, which is approximately 55 billion dollars per year. Using the average house price to be 100 thousand dollars, the number of housing starts per year is approximately 550 thousand.
[8] Experiments with various values of demand elasticity revealed that changing the values is insensitive to the calibration result, unless the value of the elasticity is either zero or infinite.

- A_1 then is approximately 0.0000731, which implies that the rental demand function is flat. Table 4 gives a summary of these values.

Using the approximated parameter values above, the roots of Equation 26 are calculated to be:

$$\{-2.44, 0.0018 + 0.99i, 0.0018 - 0.99i, 0.44\}$$

One root is explosive and three are stable. Two of the stable roots are complex conjugate so cycles in housing stocks occur in this calibrated two-period model. The more interesting question is how long are the cycles. To answer this question, various spectral densities were analysed using Laplace transform frequency response.

A visualization of cycles in capital stock is provided by spectral variance decompositions of the fourth-order system of Equation 26 in Fig. 5. The graph is generated by a function which computes the 'spectrum' (i.e. the squared amplitude of the frequency response function) of a stochastic process in Equation 26. The frequency response is generated by the Laplace Transformation (in the frequency domain). The frequency is over the entire circle. The graph reveals an important point: the accumulation of mass a little over every 2 radians (i.e. 4π) implies a cycle in housing stock of somewhere around three periods (quarters). A periodicity is defined by 2π/radian. The two-period recursive set-up of investment where the output is not realized until the beginning of the third period, indicates that the empirical mass near 2 radians for capital is reasonable. Figure 5 also shows a small mass point around 1 radian, which implies a capital cycle period of around 6.14. This longer cycle, however, is not predicted from the two-period gestation model. Even with the simple model, the cycle is the US housing industry can be replicated. These cycles are driven by the intrinsic lags in the period of production.

Table 4. *Summary of parameter values for the two-period model calibration*

Parameter	Value
Fraction of values put in place: ω_1, ω_2	0.5 and 0.5
Quarterly depreciation rate: δ	0.0035
Quarterly real interest rate: r	0.034
Elasticity of rental demand: ε_R	-1
Adjustment cost (dollar per start): $b = P/I$	0.018
Quarterly rental price in dollars: $R = P(1-\rho)$	2874
Steady state capital stock: $K = I/\delta$	3.93×10^7
Slope of rental demand: $A_1 = R/K$	$7 - 31 \times 10^{-5}$

V. CONCLUSION

A supply-determined model of the US investment in single-family dwelling housing units is investigated. The evidence suggests that models of gestation lags along with adjustment cost technology contribute towards understanding housing investment cycles and shed light on investment behaviour in general. More specifically, at least two lags are important in explaining the cycles in the quarterly housing investment. This result is numerical evidence which supports the fact that on average it takes 6.12 months to

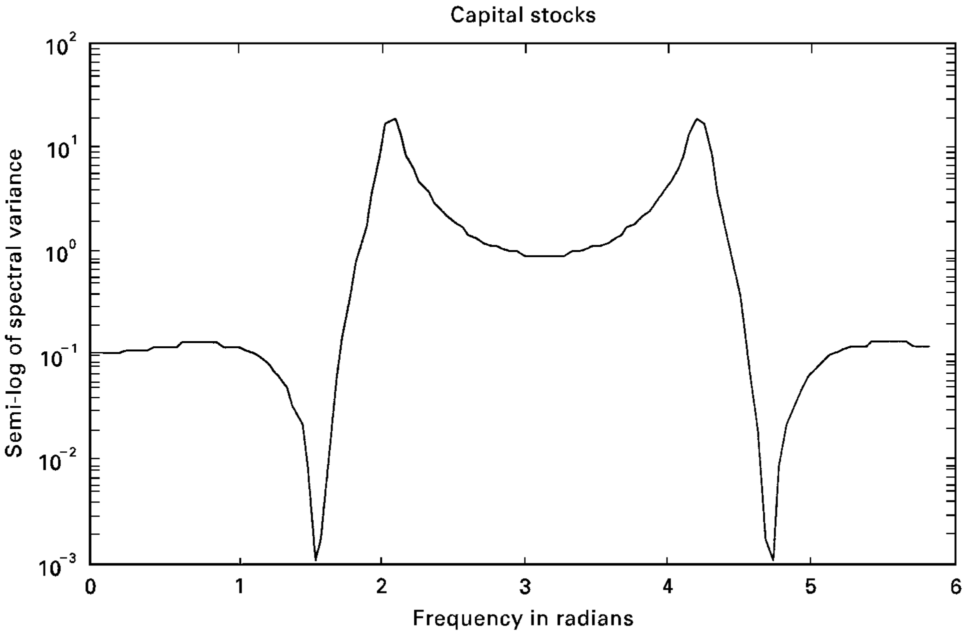

Fig. 5. *Spectral variance of capital stock with two-period lags*

complete a single-family unit house. The findings in the paper suggest that time-to-build with adjustment cost technology warrants consideration when one analyses investment functions, and provides a partial answer to the great cyclical fluctuation in housing investment. Further, the investment model in the paper shows that the inclusion of lags in empirical investment models need no longer be ad hoc.

Investment with lags, moreover, could provide one of the causes of the overbuilding 'puzzle' in the construction industry. In the proposed model, the solution to the optimization problem implies that current total gross investment depends on the current expectation that is projected on J future price and other future variables. No more than the pre-specified J lags appear between the expectation operator and the housing price. This result formalizes the stylized fact that housing investment cycles lead business cycles. That is, according to my model, a housing investor invests with respect to the future price expectation in spite of current economic conditions in order to reap the benefits from favourable future market conditions Bar-Ilan and Strange (1996) also show that under uncertainty – due to lags in investment – and for some parameter values 'optimizing investors might choose to develop in spite of unfavorable current conditions in order not to be out of the market if it improves' (p. 620).

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REFERENCES

Abel, A. B. (1980) Empirical investment equations: an integrative framework, *Carnegie-Rochester Conference Series Public Policy*, **12**, 39–91.

Abel, B. and Blanchard, O. (1986) The present value of profits and cyclical movements in investment, *Econometrica*, **54**, 249–73.

Altug, S. (1989) Time to build and aggregate fluctuations: some new evidence, *International Economic Review*, **30**, 889–920.

Bar-Ilan, A. and Strange, W. C. (1996) Investment lags, *American Economic Review*, **86**, 610–22.

Bils, M. and Kahn, J. A. (1994) The Cyclical Behavior of Inventories and Time-to-build in the New Housing Market. Working paper, University of Rochester.

Chrinko, R. (1987) Will the neoclassical theory of investment please rise? The general structure of investment models and their implications for tax policy, *The Fourth John Deutsch Roundtable on Economic Policy*, 109–167.

Christiano, L. and Todd, R. M. (1996) Time to plan and aggregate fluctuations, *Federal Reserve Bank of Minneapolis Quarterly Review*.

Hayashi, F. (1982) Tobin's marginal q and average q: a neoclassical interpretation, *Econometrica*, **50**, 213–24.

Kearl, J. R. (1979) Inflation, mortgages and housing, *Journal of Political Economy*, **87**, 1115–38.

Kling, J. L. and McCue, T. (1987) Office building investment and the macroeconomy: empirical evidence, *Journal of the American Real Estate and Urban Economics Association*, **15**, 234–55.

Kydland, F. E. and Prescott, E. C. (1982) Time to build and aggregate fluctuations, *Econometrica*, **50**, 1345–70.

Lee, G. S. (1997) Housing Investment and the Period of Production. Working paper, Institute for Advanced Studies, Vienna, Austria.

MacRae, K. M. (1989) Critical issues in electric power planning in the 1990s, **1**, Canadian Energy Research Institute, Calgary.

Mayer, T. (1960) Plant and equipment lead times, *Journal of Business*, **33**, 127–32.

Merkies, A. H. Q. M. and Steyn, I. J. (1994) Modelling changing lag patterns in Dutch construction, *Journal of Economic Dynamics and Control*, **18**, 499–509.

Montgomery, Michael. (1995) Time-to-build completion patterns for nonresidential structures, 1961–1991, *Economics Letters*, **48**, 155–63.

Mussa, M. (1977) External and internal adjustment costs and the theory of aggregate and firm investment, *Econometrica*, **44**, 163–78.

Park, J. A. (1984) Gestation Lags With Variable Plans: An Empirical Study of Aggregate Investment. Carnegie-Mellon University, Graduate School of Industrial Administration, Ph.d. Dissertation.

Poterba, J. (1984) Tax subsidies to owner-occupied housing: an asset market approach, *Quarterly Journal of Economics*, **99**, 729–52.

Poterba, J. (1991) House price dynamics: the role of tax policy and demography, *Brookings Papers on Economic Activity*, **2**, 143–83.

Rosen, S. and Topel, R. (1988) Housing investment in the United States, *Journal of Political Economy*, **96**, 718–40.

Sheffrin, S. M. (1983) *Rational Expectations*, Cambridge University Press, Cambridge.

Stein, J. (1993) Prices and Trading Volume in the Housing Market: A Model with Downpayment Effects. National Bureau of Economic Research working paper, No. 4373.

Summers, L. (1981) Taxation and corporate investment: a q-theory approach, *Brookings Papers on Economic Activity*, **1**, 67–127.

Taylor, J. (1982) The Swedish investment funds system as a stabilization rule, *Brookings Papers on Economic Activity*, **1**, 57–99.

Tobin, J. (1969) A general equilibrium approach to monetary theory, *Journal of Money Credit and Banking*, **1**, 15–29.

DATA APPENDIX

Time series data used in the paper are seasonally adjusted, quarterly from 1963 : 1 to 1991:4 with 1987 = 100 as a base year, and from the following sources.

New single-family housing prices. The price data were obtained from surveys conducted by the Bureau of the Census since 1963 for new single-family homes actually sold during the period. The index refers to characteristics of a standard 1987 quality house as obtained from a hedonic regression of actual price data on a vector of house characteristics in each year. Source: US Bureau of the Census, New One-Family

Houses Sold and for Sale (Construction Reports, ser. c25). For the Citibase code: HNPRR.

Boeckh cost index. A weighted average of construction input prices for small residential structures. Source: US Department of Commerce, Bureau of Industrial Economics, Construction Review. This series is reported bi-monthly. Linear interpolation is performed in order to obtain monthly data, and then this monthly series is transformed into a quarterly series.

Nominal interest rates. Three months US Government Treasury Bills. Source: Board of Governors of the Federal Reserve System. Also reported in the Federal Reserve Bulletin, T 1.35. For the Citibase code: FYGN3.

Mortgage interest rates. Data are combined weighted averages of interest rates on conventional first mortgage loans for the purchase of new single-family homes. They are confirmed to loans originated directly (rather than by correspondents) and are compiled from data received through the cooperation of a representative sample of four major types of lenders in the US. These lending institutions are savings and loan associations, mortgage companies, mutual savings banks, and commercial banks. Source: Office of Thrift Supervision (OTS) or the same data can be found in *Business Statistics*, The Biennial Supplement to the Survey of Current Business.

Investment. Fixed Residential Investment in single family structures. Source: US Bureau of Economic Analysis, The National Income and Product Accounts of the United States. Citibase code: GFIRSQ (units: billion dollars).

Consumption. Aggregate real consumption expenditures. Source: US Bureau of Economic Analysis, The National Income and Product Accounts of the United States, Citibase code: GDPQ (unit: billion dollars).

Months. Median number of months of new houses on market. Start to sale (number of months). Source: US Department of Commerce, Bureau of the Census. Or, US Department of Housing and Urban Development. New one-family houses sold and for sale. Also reported in Construction reports: C25. Citibase code: HNMM.

Consumer Price Index. Urban consumers price index for all items. Source: US Department of Labor, Bureau of Labor Statistics. Citibase code: PUNEW.

Energy Price Index. Urban consumers energy price index. Source: US Department of Labour, Bureau of Labor Statistics. Citibase code: PU803.

Families. The number of households. This includes the related family members and all unrelated persons who share the housing unit. Source: US Department of Commerce, Bureau of the Census, The Current Population Survey. Citibase code: POH (units: thousands).