# Effects of Securities Transaction Taxes on Depth and Bid-Ask Spread[1]

Dominique Y. Dupont
University of Twente
School of Management and Technology
Department of Finance and Accounting
Postbus 217, 7500 AE Enschede,The Netherlands
Tel: +31 53 489 44 70, Fax: +31 53 489 21 59
Email: d.dupont@sms.utwente.nl

Gabriel S. Lee[2]
Institute for Advanced Studies
Department of Economics and Finance
Stumpergasse 56
A-1060 Vienna, Austria
Tel: +43.1.59991.141, Fax: +43.1.59991.555
Email: gabriel.lee@ihs.ac.at

July, 2003
JEL: G14, D82
Key words: Asymmetric Information, Securities Transaction Taxes, Liquidity

[1]We thank Vincent Reinhart, Walter Fisher and seminar participant at the Institute for Advanced Studies, Vienna for helpful discussions. This paper was partially written when Dominique Dupont was at the Board of Governors of the Federal Reserve System. The usual disclaimer applies.
[2]Corresponding author.

## Abstract

This paper investigates the effects of transaction taxes on depth and bid-ask spread under asymmetric information. The paper uses a static model where a monopolistic market maker faces liquidity and informed traders. Introducing transaction taxes could, surprisingly, lead to increase in depth. Under some distributional assumptions, when market conditions are favorable to the dealer (i.e. when information asymmetry is weak or liquidity demand is strong), the spread responds less than proportionally to an increase in the transaction tax while the depth actually increases. In contrast, when market conditions are unfavorable to the dealer, the spread widens more than proportionally and the depth decreases, potentially to zero, in response to an increase in the transaction tax.

## 1. Introduction

This paper investigates the effects of securities transaction taxes (STT) on the liquidity of a quote-driven market under asymmetric information. Academics and financial market regulators (e.g. [4] , [9] and [7]) have studied the potential effects of imposing securities transaction taxes (akin to the Tobin tax) as an instrument to curb speculation and excess volatility without impairing market liquidity. [8] states that liquidity can be measured by bid-ask spread, depth and resiliency.[1] Past research, however, uses an aggregate measure of liquidity, trading volume, in analysing the relationship between transaction taxs and liquidity. In this paper, we examine the effects of taxation on a disaggregate level using bid-ask spread and depth.[2] Consequently, our results can accommodate some of the disagreements in the empirical literature on the relative magnitude of transaction taxes on trading volume ([1] and [2]).

Our model builds on [6], who study the pricing strategy of an uninformed market maker facing potentially better informed traders. In contrast to [6], however, we incorporate both spread and depth into a model that allows the dealer to adjust the depth differently than the bid-ask spread in response to changes in the degree of information asymmetry (see also [5]). In this setting, we analyse the effects of transaction tax on market liquidity across different levels of information asymmetry. Our analysis uses a one-period model where a monopolistic market maker posts firm prices (including tax) and depths on the bid and ask sides, while facing a risk-neutral informed trader and a liquidity trader. The informed trader observes a private signal correlated with the true value of the asset. The demand of the liquidity trader is price sensitive

[1] Resiliency is the speed with which price fluctuations resulting from trades are dissipated, and depth is the maximum amount the dealer stands ready to sell or buy at the posted prices. See [8] for details.

[2] Previous research on the relationship between transaction costs and trading volume can be used to study the effects of transaction taxes on liquidity (see among many, [3], [1], [10], and [11] .

and subject to the liquidity shock.

Our results can be summarized as follows. First, introducing a transaction tax could lead to either an increase or a decrease in depth, depending on the degree of information asymmetry and liquidity demand. Secondly, the spread could respond disproportionally to increase in tax. Subsequently, our results seem to point to two regimes as far as transaction tax is concerned. First, when information asymmetry is weak or liquidity demand is strong (i.e.,when market conditions are favorable to the market maker), the market maker pays part of the transaction tax himself by increasing the spread less than the tax. Moreover, he quotes a larger depth to attract order flow in order to make up for the loss in demand due to the transaction tax. On the other hand, when market conditions are unfavorable, increasing the transaction tax leads to a drastic reduction in the liquidity provided by the market maker, enticing him to exit the market. In turn, the paper suggests that lowering transaction tax may not necessarily lead to increase in quoted depth.

Our paper is structured as follows. The next section introduces the model. To analyse our model under various distributional assumptions, we study two cases. Section 3 analyses the equilibrium conditions under a discrete distribution: the asset value, the informed trader's signal and the liquidity shock each take two values. Section 4 looks at the equilibrium conditions under continuous distributions where the asset value is lognormally distributed with mean 1. Both the private signal and the liquidity shock are, however, normally distributed. Section 5 concludes the paper.

## 2. Model

A monopolistic dealer posts firm prices and depths on the bid and ask sides. He faces a price-sensitive liquidity trader and a trader possessing private information about the value of the asset. The liquidity trader's demand is price sensitive and stochastic. The informed trader, who observes a signal correlated with the true value of the asset, buys the asset if the ask price is below (or sells if the bid price is above) his valuation. For simplicity, we assume no limit-order book or floor brokers compete with the dealer (or specialist). The model focuses on asymmetric information and excludes other factors such as misdiversification and order-processing costs.

To simplify computations, we assume that the informed trader is risk neutral. Informed and liquidity traders' buy or sell orders are lumped together and passed on to the dealer so that the quantity limit becomes binding if the sum of all orders is greater than the posted depth.

We study only the ask side of the dealer's activity as the bid side is symmetrical. Let $x$ be the true value of the asset, $c$ the transaction tax, $a$ the ask price inclusive of tax, $z$ the quantity limit, $G$ the informed trader's private signal, $v = E[x|G]$ his valuation of the traded asset, $\eta$ the liquidity shock ($\eta$ and $G$ are independently distributed), and $d(a)$ the liquidity trader's demand.

The orders of the liquidity trader and of the informed trader are pooled together and passed on to the dealer. By convention, the dealer pays the transaction tax but, ceteris paribus, the transaction tax could be levied on the customers. To acquire one unit of the traded asset, informed and liquidity traders pay $a$ but the dealer receives only $(a - c)$ if the transaction tax is a fixed amount per transaction (fixed transaction tax), or $(\frac{a}{1+c})$ if it is a fixed percentage of the price (proportional transaction tax).

The effective demand, $S(a, z, v, \eta)$, depends on the realization of liquidity shocks and the valuation of informed trader. If $v > a$ the informed trader asks the maximum amount; if $v \leq a$ the informed trader does not participate and the transaction volume on the ask side depends on the liquidity demand. The following equation represents the effective demand:

$$S(a, z, v, \eta) = I(v \leq a)\ I(d(a) \geq 0)\ \min(d(a), z) + I(v > a)\ z \tag{1}$$

where $I$ the indicator function. The dealer's realized profit and expected profit are given in the following two equations:

$$\Pi = \left\{ \begin{array}{c} S(a, z, v, \eta)\,(a - c - x)\ \text{ for the linear tax} \\ \\ S(a, z, v, \eta)\left(a - \frac{c}{1+c} - x\right)\ \text{ for the proportional tax} \end{array} \right\} \tag{2}$$

$$E[\pi] = \left\{ \begin{array}{c} E[I[v \leq a](a - c - v)]\,E[I[d(a) \geq 0]\min(d(a), z)] \\ +E[I[v > a](a - c - v)]\,z \text{ for the linear tax} \\ \\ E[I[v \leq a](a/(1 + c) - v)]\,E[I[d(a) \geq 0]\min(d(a), z)] \\ +E[I[v > a](a/(1 + c) - v)]\,z \text{ for the proportional tax} \end{array} \right\} \tag{3}$$

## 3. Equilibrium with discrete distributions

Let $x$ and $G$ take the values 1 or $-1$ and $\eta$ take the values $\bar{\eta}$ or $-\bar{\eta}$ with probability $\frac{1}{2}$. The demand then becomes $d(a) = -a + \eta$. The informed trader's valuation $v$ takes values $\bar{v} = E[x|G = 1]$ and $\underline{v} = E[x|G = -1] = -\bar{v}$ with probability $\frac{1}{2}$. $\bar{v}$ is in between 0 and 1 and increases with the precision of the information of the informed trader. In the limit, when the latter is perfectly informed, that is, when he observes the realization of $x$, $\bar{v} = 1$. The tax is linear. We assume that $c < \frac{2}{3}\bar{\eta}$. We also impose $a \leq \bar{\eta}$ since setting $a$ above $\bar{\eta}$ would make the liquidity demand always negative and the liquidity trader would not trade in this case.

The market maker can take the following strategies:

1. The market maker can exclude the informed trader by setting the selling price above the latter's maximum valuation: $a \geq \bar{v}$. Facing no informed trader, the dealer imposes no quantity limit. Consequently, his expected profit is

$$E[\pi] = E\left[ I[d(a) \geq 0]\, d(a) \right] (a - c) = E\left[ I[\eta \geq a]\, (\eta - a) \right] (a - c) = \frac{1}{2}\, (\bar{\eta} - a)\, (a - c). \quad (4)$$

   The profit-maximizing price, $(c + \bar{\eta})/2$ is the listed price provided this price is above $\bar{v}$ and the maximum value for the profit is $\frac{1}{8}(\bar{\eta} - c)^2$. To summarize, the optimal price that excludes the informed trader is $\max[\bar{v}, (c + \bar{\eta})/2]$; there is no quantity limit; the maximum expected profit is $\frac{1}{8}(\bar{\eta} - c)^2$ or $\frac{1}{2}\,(\bar{v} - c)\,(\bar{\eta} - \bar{v})$ depending on whether $(\bar{\eta} + c)$ is above or below $\bar{v}$.

2. Alternatively, the market maker can set his selling price below $\bar{v}$ in the hope of attracting more demand; $0 < a < \bar{v}$.[3] Naturally, he then runs the risk of being picked by the informed trader. Given a selling price $a$, $\bar{\eta} - a$ is the highest possible liquidity demand. If the dealer sets the depth strictly above this level, he would incur extra losses when transacting with the informed trader without generating extra liquidity trade. Hence, the dealer sets the

[3] If $0 < a < \bar{v}$ then $I[v \leq a] = I[v = -\bar{v}]$, $I[v > a] = I[v = \bar{v}]$.

depth at or below $\bar{\eta} - a$ and his expected profit is:

$$\begin{aligned}
E[\pi] &= E[I[v \leq a](a-c-v)]\, E[I[d(a) \geq 0]\, z] + E[I[v > a](a-c-v)]\, z \\
&= \Big( E[I[v \leq a](a-c-v)]\, p(d(a) \geq 0) + E[I[v > a](a-c-v)] \Big)\, z \\
&= \Big( \tfrac{1}{4}\,(a+\bar{v}-c) + \tfrac{1}{2}\,(a-\bar{v}-c) \Big)\, z \\
&= \tfrac{3}{4} z (a - c - \tfrac{1}{3}\bar{v})
\end{aligned} \tag{5}$$

If $a < c + \bar{v}/3$, the optimal $z$ is 0; if $a > c + \bar{v}/3$, the optimal $z$ is $\bar{\eta} - a$; if $a = c + \bar{v}/3$, $z$ can be set arbitrarily, for example, to zero.

With $z = \bar{\eta} - a$, the expected profit function becomes:

$$E[\pi] = \frac{3}{4}\,(\bar{\eta} - a)\,(a - c - \frac{1}{3}\,\bar{v}). \tag{6}$$

The dealer maximizes the function defined in Equation (6) under the constraint that $0 < a < \bar{v}$. If $\bar{v} > 3\,(\bar{\eta} - c)$, this function is negative for $a < \bar{\eta}$ and the dealer exits the market. If $\bar{v} \leq 3\,(\bar{\eta} - c)$, the price that maximizes $E[\pi]$ without the constraint $a < \bar{v}$, $\frac{1}{2}(c + \bar{\eta}) + \frac{1}{6}\bar{v}$, is smaller than $\bar{v}$ if $\bar{v} > \frac{3}{5}\,(\bar{\eta} + c)$. Hence, if $\frac{3}{5}\,(c + \bar{\eta}) < \bar{v} \leq 3\,(\bar{\eta} - c)$, the optimal price is $\frac{1}{2}\,(\bar{\eta} + c) + \frac{1}{6}\bar{v}$ and the maximum profit is $\frac{1}{48}\,(3\bar{\eta} - 3c - \bar{v})^2$. If $\bar{v} \leq \frac{3}{5}(c + \bar{\eta})$, the constraint $a < \bar{v}$ is binding; in the limit, the dealer wants to set $a = \bar{v}$.

The final step is to compare the maximum expected profit when $a > \bar{v}$ and when $a \leq \bar{v}$. When $\frac{3}{5}(\bar{\eta} + c) < \bar{v} < 3\,(\bar{\eta} - c)$, maximizing expected profit setting $a \geq \bar{v}$ would yield a lower maximum than letting $a < \bar{v}$. The optimal price and depth in this case are $a^* = \frac{1}{2}(\bar{\eta} + c) + \frac{1}{6}\bar{v}$ and $z^* = \bar{\eta} - a^*$. When $\frac{1}{2}(\bar{\eta} + c) < \bar{v} \leq \frac{3}{5}(\bar{\eta} + c)$, the optimal price is $\bar{v}$ and no quantity limit is imposed. When $\bar{v} \geq 3(\bar{\eta} - c)$, the depth is set to zero and no price need be quoted. The above results are summarized in Table 1.

Insert Table 1 About Here.

Figure 1 maps the results of Table 1. It plots the half spread, $a$, and the depth, $z$, against $\bar{v}$. The main message from Figure 1 is that the introducing of a transaction tax has non-linear effects on the spread and the depth. Imposing a finite depth is not necessary if the asymmetry in information is low enough. When $\bar{v}$ is low, a small rise in the transaction tax only results in an increase in the spread while a larger change may also bring about the imposition of a finite quantity limit. A small increase in the transaction tax may not even affect the selling price when the latter is set exactly equal to the informed trader's high valuation. In contrast, when the information asymmetry is more severe, a higher transaction tax could lead the dealer to completely exit the market.

Insert Figure 1 About Here.

## 4. Equilibrium with continuous distributions

In the following section, we take the asset value, $x$, to be lognormally distributed and the demand to be $d(a) = -\log(a) + \eta$. We also assume that $y, \eta$ and $G$ are jointly normally distributed where $y = \log(x)$.

### 4.1. Marginal condition

Although no equilibrium closed-form solution exists, the marginal conditions for the price and the quantity limit shed light on the trade-offs governing the dealer's profit maximization.

The first derivative of the specialist's expected profit with respect to the quantity limit is given in Equation (7); the one with respect to the ask price is given in Equation (8), where $d'(a)$ denotes the slope of the liquidity demand at price $a$.[4]

$$\frac{\partial}{\partial z}E[\pi] = p(d(a) > z)\ E\Big[I[v \leq a]\ (a - c - v)\Big] + E\Big[I[v > a]\ (a - c - v)\Big], \tag{7}$$

$$\begin{aligned} \tfrac{\partial}{\partial a}E[\pi] &= p(v > a)\ z + p(v \leq a)\ E\Big[\ I[d(a) \geq 0]\ \min(d(a), z)\Big] \\ &+ d'(a)\ p(0 \leq d(a) \leq z)\ E\Big[\ I[v \leq a]\ (a - c - v)\Big]. \end{aligned} \tag{8}$$

Equation (7) has an intuitive interpretation. Increasing the quantity limit by an infinitesimal amount brings the market maker a profit equal to $E\Big[\ I[v \leq a]\ (a - c - v)\Big]$ if the limit is hit by a liquidity trader, or a loss equal to $E\Big[\ I[v > a]\ (a - c - v)\Big]$ if it is hit by an informed trader. Equation (7) is similar to the standard monopolistic case in which increasing the price by \$1 makes the monopolist earn another \$1 on the total volume but reduces the overall demand. The first two terms on the right-hand side of Equation (8) represent the expected sales when the informed trader wants to trade, $p(v > a)\ z$, and when he does not, $p(v \leq a)\ E\Big[\ I[d(a) \geq 0]\ \min(d(a), z)\Big]$. The last term represents the impact on profits resulting from the reduction in overall demand. A price increase depresses demand if the quantity limit is not binding, that is, if the liquidity trader's demand is positive but lower than $z$ and the informed trader's evaluation is below $a$. In this event, the effect on overall demand is proportional to the slope of the liquidity trader's demand.

[4] The derivation of equations (7) and (8) can be provided upon request.

### 4.2. Numerical Results

When $x$ is lognormally distributed, we search for the optimal price (or spread) and for the optimal depth numerically. Figure 2 graphs the half-spread and depth, both divided by $\sigma_\eta$, against $\rho\sigma_y/\sigma_\eta$ when $x$ is lognormally distributed and $\sigma_\eta = 0.1$.[5] As in the discrete case, introducing a transaction tax has non-linear effects on the spread and the depth and the effect depends on whether market conditions are favorable or unfavorable to the dealer.

The spread increases and the depth decreases with $\rho\sigma_y/\sigma_\eta$. When market conditions are very favorable - that is, the informed trader is poorly informed - the depth grows without bound while the spread remains finite because of the dealer's market power. When market conditions become very unfavorable, the dealer exits the market by setting the depth to 0 while maintaining a finite spread. When $x$ is lognormally distributed, half-spread and depth are approximately proportional to $\sigma_\eta$. For example, for $\rho\sigma_y/\sigma_\eta = 0.5$, the half-spread equals about 0.8 percent when $\sigma_\eta = 0.01$ and 9 percent when $\sigma_\eta = 0.1$.

When market conditions are favorable to the dealer (e.g. $\rho\sigma_y/\sigma_\eta \in (0.2, 0.4)$), introducing a transaction tax increases the spread by less than the amount of the transaction tax and creates a larger depth. On the other hand, when market conditions are unfavorable to the dealer (e.g. $\rho\sigma_y/\sigma_\eta \in (0.8, 1.0)$), introducing a transaction tax pushes the price up by more than the transaction tax and narrows the depth.

Insert Figure 2 About Here.

[5] $\sigma_y$ is the standard deviation of $y = \log(x)$, $\sigma_\eta$ is the standard deviation of $\eta$ and $\rho = corr(G, y)$. Note that $\overline{v} = \rho\sigma_y$ and $\overline{\eta} = \sigma_\eta$. Consequently, an increase in $\rho\sigma_y/\sigma_\eta$ represents an increase in the degree of asymmetric information: the valuation of informed trader increase, which translates into an unfavorable market condition for the dealer.

### 4.3. Application: the US Treasury market

In both the discrete and lognormal cases, the strength of the liquidity demand plays a central role in determining the equilibrium spread and depth. When studying the market marking process, one can focus on the liquidity conditions the dealer faces rather than on information asymmetry. This is particularly suited to the U.S. government bond market. For example, spreads are typically narrower and depth larger for securities that are the most recent issues in their maturity class (the on-the-run Treasuries) than for similar securities issued just before (the off-the-run Treasuries). Bid-ask spreads on coupon Treasury securities have traditionally been about twice as high for off-the-run issues than for comparable on-the-run issues, while quoted depth is lower. Moreover, during the recent bouts of market volatility, bid-ask spreads have widened and depth has contracted proportionally more for off-the-run than for on-the-run coupon securities. The asymmetric information argument cannot easily account for this fact since the quality of private information should be equal across the two market segments. However, agents trading bonds prefer to trade in the on-the-run securities, therefore creating a stronger liquidity demand in that market. The more fundamental question as to why agents prefer to trade in the on-the-run segment is not addressed here, although self-fulfilling expectation arguments could be made.

### 4.4. Two regimes

The paper points to two regimes as far as transaction taxes are concerned. When market conditions are favorable, the dealer pays part of the tax himself (by increasing the spread less than the tax) and quotes a larger depth to attract order flow in order to make up for the loss in demand due to the transaction tax. The increase in the depth offsets, albeit partially, the effect on trading volume of the wider spread. This is because, when market conditions are rather favorable to the dealer, a tax—insofar as it is at least partially reflected in the bid and ask prices—reduces the probability of the informed trader's buying at the ask or selling at the bid. This additional protection entices the market maker to quote a larger depth than he would without tax. When market conditions are unfavorable, increasing the transaction tax leads to a drastic reduction in the liquidity provided by the market maker, enticing him to exit the market.

Although, the implications of the model have been introduced by presenting the effect on the spread and the depth of an increase in the transaction tax, symmetric conclusions hold for a reduction in this tax. As a consequence, a decision to lower taxes on transactions in the hope of improving market liquidity might actually lead to smaller depths and a less-than-proportional reduction in the spreads.

## 5. Conclusion

The model shows that introducing a transaction tax could affect market liquidity differently depending on the market conditions facing the dealer. If the asset value, the informed trader's signal, and the liquidity shock each take two values, there is an interval for the precision of the private signal for which increasing the transaction tax has no or little effect on the spread (no quantity limit is necessary then). If the asset value is lognormally distributed, the market maker widens the spread by less than the transaction tax and actually increases the depth when the degree of information asymmetry is subdued or liquidity demand is strong. In contrast, when market conditions are unfavorable to the dealer, he increases the spread by more than the transaction tax and reduces the depth. In all cases, introducing a transaction tax may induce the dealer to exit the market before he would have done so in the absence of a transaction tax. This suggests that a transaction tax could aggravate liquidity loss in periods of market stress.

## References


[1] Barclay, M.J., Kandel, E. and Marx, L.M. ,: The Effects of Transaction Costs on Stock Prices and Trading Volume. Journal of Financial Intermediation **7**, 130-50 (1998)

[2] Brennan, M.J., Chordia, T., and Subrahmanyam, A.: Alternative Factor Specifications, Security Characteristics, and the Cross-Section of Expected Stock Returns. Journal of Financial Economics **49** 345-73 (1998)

[3] Constantinides, G.M.: Capital Market Equilibrium with Transaction Costs. Journal of Political Economy **94**, 842-62 (1986)

[4] Dow, J. and Rahi, R.: Should Speculators Be Taxed. Journal of Business **73**, 89-107 (2000)

[5] Dupont, D.: Market making, prices, and quantity limits. Review of Financial Studies **13**, 1129-1151 (2000)

[6] Glosten, L. and Milgrom, P.: Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders. Journal of Financial Economics **14**, 71-100 (1985)

[7] Hakkio, C.S.: Should we throw sand in the gears of financial markets. Federal Reserve Bank of Kansas City, Economic Review 79, 17-30 (1994)

[8] Kyle, A.: Continuous Auctions and Insider Trading. Econometrica **53**, 1315-1335 (1985)

[9] OECD.: Exchange market volatility and securities transaction taxes. OECD Economic Outlook **71**, 185-197 2002

[10] Vayanos, D.:Transaction Costs and Asset Prices: A Dynamic Equilibrium Model. Review of Financial Studies **11**, 1-58 (1998)

[11] Vayanos, D. and Vila J.L.: Equilibrium Interest Rate and Liquidity Premium with Transaction Costs. Economic Theory **13**, 509-39 (1999)

Table 1: Equilibrium price and quantity limit with risk-neutral informed trader and discrete distributions

| $\psi$ | $\psi \le \frac{1}{2}(\mu + c)$ | $\frac{1}{2}(\mu + c) < \psi \le \frac{3}{5}(\mu + c)$ | $\frac{3}{5}(\mu + c) < \psi \le 3(\mu - c)$ | $\psi > 3(\mu - c)$ |
|---|---|---|---|---|
| optimal $a$ | $\frac{1}{2}(\mu + c)$ | $\psi$ | $\frac{1}{2}\mu + \frac{1}{2}c + \frac{1}{6}\psi$ | n.a. |
| optimal $z$ | n.a. | n.a. | $\frac{1}{2}\mu - \frac{1}{2}c - \frac{1}{6}\psi$ | 0 |
| $\max(E[\pi])$ | $\frac{1}{8}(\mu - c)^2$ | $\frac{1}{2}(\psi - c)(\mu - \psi)$ | $\frac{1}{48}(3\mu - 3c - \psi)^2$ | 0 |
| maximum $c$ | $\mu$ | $\frac{2}{3}\mu$ | $\frac{1}{4}\mu$ | 0 |

When $a \ge \psi$, imposing a quantity limit is not necessary; the optimal $z$ is undefined and "n.a." is entered in the corresponding cell. When $\psi > 3(\mu - c)$, the expected profit is never positive and the dealer exits the market by setting $z$ to $0$; the optimal $a$ is undefined and "n.a." is entered in the corresponding cell.

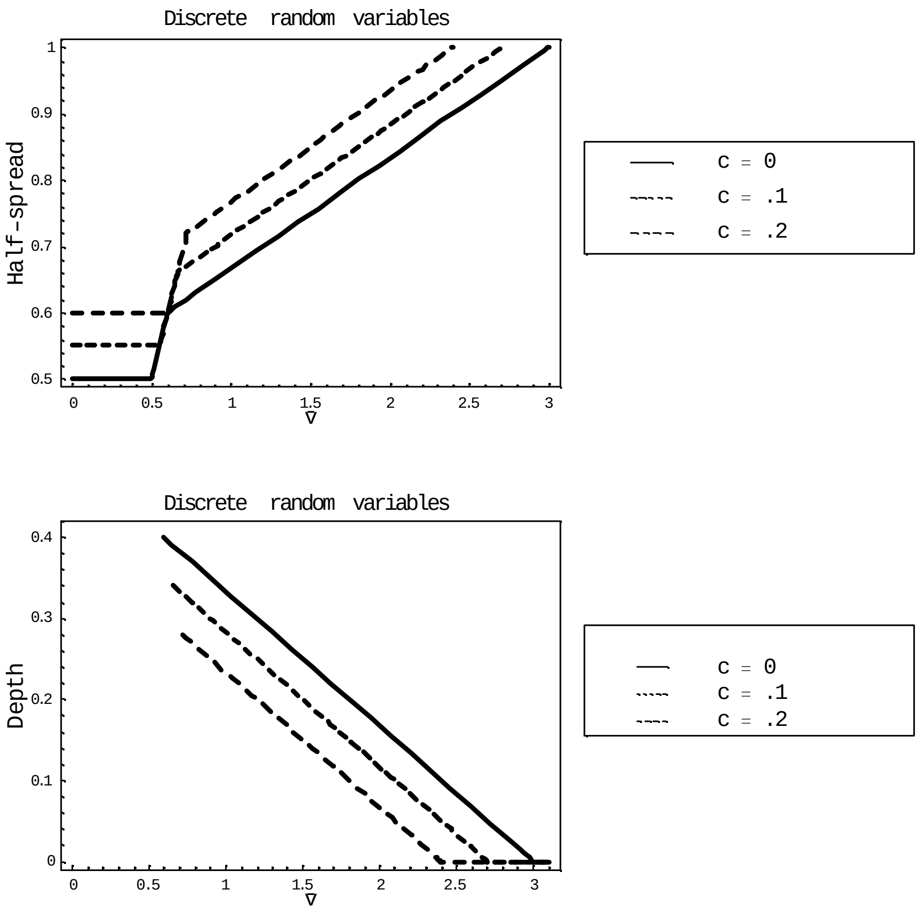


Figure 1: Half-spread and depth when the informed trader's valuation and the liquidty shock each take two values with probability 1/2. The positive value of the liquidity shock is 1, that of the informed trader's valuation is $\overline{v}$ and is displayed on the horizontal axis. This …gure represents the market conditions facing the dealer.

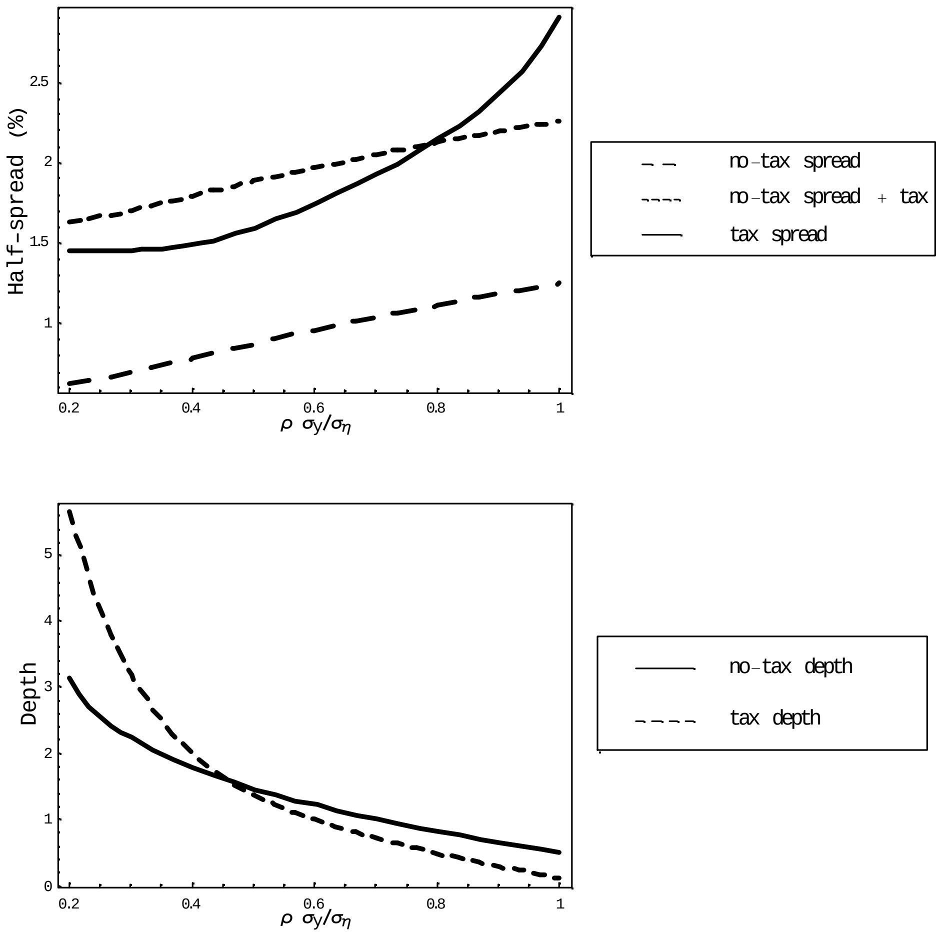


Figure 2: Half-spread (in percentage) and depth with a proportional transaction cost of 1 percent (solid line) and without transaction cost (short-dashed line). The asset value, X, is lognormally distributed and y = log(X). In the top panel, the long-dashed line is the sum of the price without transaction cost and the transaction cost. Half-spread and depth are divided by ¾´.